

# Edge Detection Combining Wavelet Transform and Canny Operator Based on Fusion Rules

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# Outline

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# 1. Introduction

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Edge detection plays an important role in computer vision and image analysis.

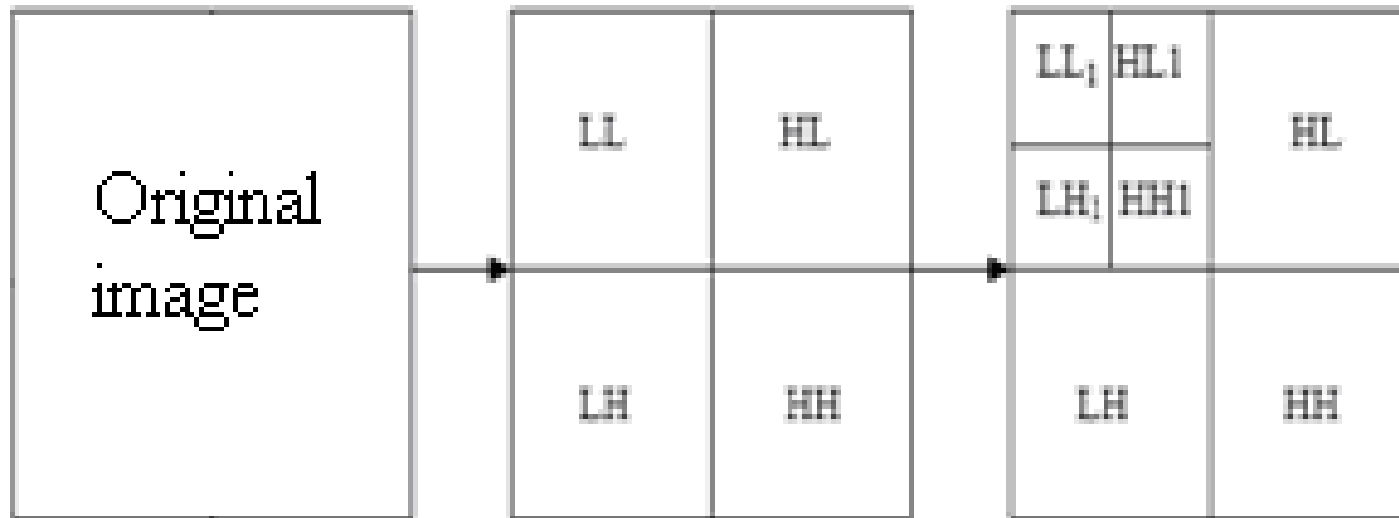
Edges are the abrupt change points in the image which are the basic features of the image. These abrupt change points give the locations of the image contour that shows the important feature .

The edge representation of an image reduces the amount of data to be processed, and it retains important information about the shapes of objects in the scene. The description of an image is easy to integrate into a large number of recognition algorithms used in computer vision and other image processing applications.

# 1. Introduction

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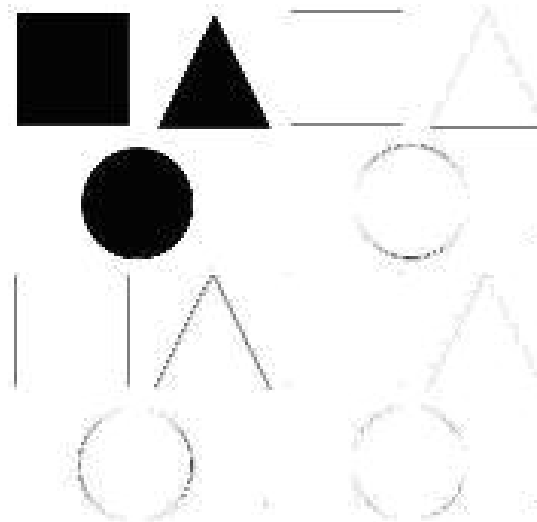
Wavelet analysis developed rapidly as a useful research method, the detection method based on multi-scale wavelet analysis is one of the new edge detection methods.



# 1. Introduction

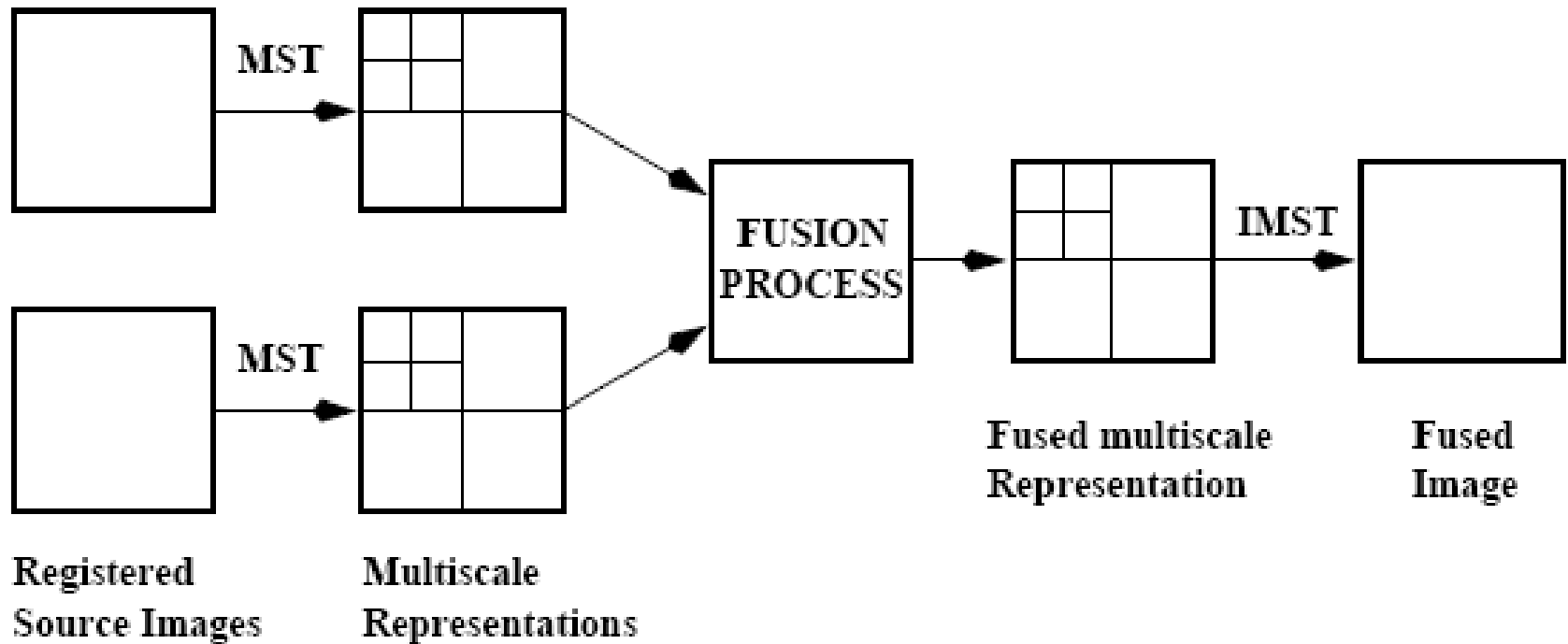
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LL shows the smoothing image of the original image which contains the most information of the original image. LH preserves the vertical edge details. HL preserves the horizontal edge details. HH preserves the diagonal details which are influenced by noise greatly.



# 1. Introduction

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## 2. Low-frequency sub-image edge detecting based on canny operator

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### 2.1 Image smoothing

$$I^*(x, y) = \frac{1}{5 \times 5} \sum_{i=-2}^2 \sum_{j=-2}^2 I(x+i, y+j) \times M(2+i, 2+j)$$
$$x=1,2,\dots,m; y=1,2,\dots,n \quad (1)$$

Where the pixel value of the low-frequency sub-image is  $I(x, y)$ ,  $M$  is Gaussian template, the pixel value of the smoothed image is  $I^*(x, y)$ .

## 2. Low-frequency sub-image edge detecting based on canny operator

### 2.2 Computing gradient direction and amplitude

Computing gradient direction and amplitude of smoothed image  $I^*(x,y)$  adopting first order partial finite difference of  $2 \times 2$  neighborhood.

$$M(x, y) = \sqrt{g_x^2(x, y) + g_y^2(x, y)} \quad (2)$$

$$Q(x, y) = \arctan[ g_x(x, y), g_y(x, y) ] \quad (3)$$

$$f_x = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \quad f_y = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \quad (4)$$

Where  $g_x$  and  $g_y$  are the results of the smoothing image filtered along rows and columns.



## **2. Low-frequency sub-image edge detecting based on canny operator**

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### **2.3 Gradient image with non-maximum suppression**

If the gradient amplitude of the pixel is no less than the gradient amplitude between two adjacent pixels in the gradient direction, the point can be judged as the edge point possibly.

## 2. Low-frequency sub-image edge detecting based on canny operator

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### 2.4 Dual threshold method of detection and connecting the edges

Selecting two thresholds  $H_{th}$  which stands for the high threshold and  $L_{th}$  which stands for the low threshold to process the gradient image. Where  $H_{th} = L_{th} * h$ ,  $h$  equals 1.5 to 2.

There will be two detecting results. Connecting the edge contour from the former and finding weak edge points from the latter to recoup the former edge gaps when connecting to the endpoint.

### **3. High-frequency sub-image edge detecting based on wavelet transform**

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#### **3.1 Denoising algorithm of the high-frequency sub-image based on wavelet transform**

When detect the edges from the high-frequency sub-images, the result will be affected by the noise.

Those different wavelet coefficients should be multiplied by a denoising factor which is relative to their own wavelet coefficients. The denoising factor is less than 1 and will decrease with the absolute values of the wavelet coefficients.

### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.1 Denoising algorithm of the high-frequency sub-image based on wavelet transform

➤ The realization process is as follows:

$$F(x, y) = \begin{cases} w(x, y) & |w(x, y)| \geq 3\sigma \\ 0 & |w(x, y)| \leq |aver| \\ w(x, y) \times k & else \end{cases} \quad (5)$$

Where  $w(x, y)$  stands for the high-frequency coefficient which is gained through wavelet decomposition,  $F(x, y)$  stands for the high-frequency coefficient gained after denoising;  $\sigma$ ,  $aver$  indicate the variance and the mean value of the high-frequency coefficient from different wavelet decomposition levels and different directions.

### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.1 Denoising algorithm of the high-frequency sub-image based on wavelet transform

$k$  is a function which is relative to the index

$$k = e^{-aw(x,y)+b} - 1 \quad (6)$$

When  $w(x, y)$  is greater than  $3\sigma$ ,  $w(x, y)$  can be considered as consisting of the signal fully. So  $k=1$ , it will be:

$$e^{(-3\sigma a+b)} - 1 = 1 \quad (7)$$

When  $w(x, y)$  is not greater than *aver*,  $w(x, y)$  approaches 0, it will be:

$$e^{-a \times \text{aver} + b} - 1 = 0 \quad (8)$$

### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.1 Denoising algorithm of the high-frequency sub-image based on wavelet transform

There will be obtained according to two upper equations:

$$a = \frac{-\ln 2}{3\sigma - aver} \quad b = a \times aver \quad (9)$$

Denoising factor  $k$  will be obtained when  $a$  and  $b$  substituted into the equation (6).

$$k = e^{-\frac{\ln 2}{3\sigma - aver} \times w(x,y) + \frac{-\ln 2}{3\sigma - aver} \times aver} - 1 \quad (10)$$

Denoising algorithm will be obtained when  $k$  substituted into equation (5).

### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.2 Edge detecting of denoising high-frequency sub-image based on wavelet transform

- After eliminating noise of the high-frequency sub-images, the edges of the high-frequency sub-images are detected using wavelet modulus maxima algorithm.
- The realization process of the algorithm is as follows.

$\theta(x, y)$  stands for Gaussian smoothing function, supposing:

$$\theta_s(x, y) = \frac{1}{s^2} \theta\left[\frac{x}{s}, \frac{y}{s}\right] \quad (11)$$

### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.2 Edge detecting of denoising high-frequency sub-image based on wavelet transform

➤ Calculating the partial derivative of the smoothing function, the wavelet function will be:

$$\phi_s^1(x, y) = \frac{\partial \theta(x, y)}{\partial x} = \frac{1}{s^2} \phi^1 \left[ \frac{x}{s}, \frac{y}{s} \right] \quad (12)$$

$$\phi_s^2(x, y) = \frac{\partial \theta(x, y)}{\partial y} = \frac{1}{s^2} \phi^2 \left[ \frac{x}{s}, \frac{y}{s} \right] \quad (13)$$



### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.2 Edge detecting of denoising high-frequency sub-image based on wavelet transform

➤ Convolution of  $f(x, y)$  will obtain two components of two dimensional wavelet transform in scale  $s$ :

$$W_s^1 f(x, y) = f * \phi_s^1(x, y) \quad (14)$$

$$W_s^2 f(x, y) = f * \phi_s^2(x, y) \quad (15)$$

### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.2 Edge detecting of denoising high-frequency sub-image based on wavelet transform

The gradient module in scale  $s$  is:

$$M_s f(x, y) = \sqrt{|W_s^1 f(x, y)|^2 + |W_s^2 f(x, y)|^2} \quad (16)$$

The angel is:

$$A_s f(x, y) = \arctan \left[ \frac{W_s^2 f(x, y)}{W_s^1 f(x, y)} \right] \quad (17)$$

### 3. High-frequency sub-image edge detecting based on wavelet transform

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#### 3.2 Edge detecting of denoising high-frequency sub-image based on wavelet transform

➤ Computing the local modulus maxima of three sub-images after wavelet transform using equation (16) and (17), then their edge images  $G_{LH}$ 、 $G_{HL}$  and  $G_{HH}$  will be obtained. Those high-frequency sub-images contain the details of the original image, and then three edge images are computed with weighting fusion rules. Computing formula is described as follows:

$$D_H(i, j) = r_{LH} * D_{LH}(i, j) + r_{HL} * D_{HL}(i, j) + r_{HH} * D_{HH}(i, j)$$

### 3. High-frequency sub-image edge detecting based on wavelet transform

#### 3.2 Edge detecting of denoising high-frequency sub-image based on wavelet transform

➤ Where  $D_{LH}(i, j)$ 、 $D_{HL}(i, j)$ 、 $D_{HH}(i, j)$  indicate the wavelet coefficients corresponding to the edge images  $G_{LH}$ 、 $G_{HL}$  and  $G_{HH}$ .  $D_H(i, j)$  stands for the wavelet coefficient after fusion,  $r_{LH}$ 、 $r_{HL}$  and  $r_{HH}$  indicate the corresponding weights. The sum of three weights is 1.

After these processes, we can get the edges of the low-frequency sub-image and the weighting edges of the high-frequency sub-images. The final edge images are obtained through wavelet composition from the fusion edge sub-images.

## 4. Experimental results

**Fig.2 Detecting result of the 'Lena' image**

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**(a) The original image**

**(b) Detecting result  
by wavelet algorithm**

**(c) Detecting result  
by improved algorithm**

## 4. Experimental results

**Fig.3 Detecting result of the tire image**

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**(a) The original image**

**(b) Detecting result  
by wavelet algorithm**

**(c) Detecting result  
by improved algorithm**

## 4. Experimental results

Figure.4 Edge detecting result of the noisy image Lena



(a) The original image

(b) Detecting result  
by wavelet algorithm

(c) Detecting result  
by improved algorithm

## 5. Conclusion

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- This paper proposed a new edge detection fusion algorithm based on wavelet transform and canny operator to detect the edge.
- The proposed method can not only get rid of the noise effectively but also stand out the image detail edges and locate the edge accurately.



# References

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**THANK YOU VERY MUCH!**