Lower Bounds for the Hadamard Maximal Determinant Problem

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Abstract

Gene Golub was interested in both matrix computations and statistics. In this Golub memorial lecture I will consider a problem that involves aspects of both – the *Hadamard maximal determinant problem*.

The problem is to find the maximal determinant of an $n \times n$ matrix whose elements are in $[-1, 1]$. A matrix achieving the maximum is known as a *D-optimal design* and has applications in the design of experiments. Hadamard proved an upper bound $n^{n/2}$ on the determinant, but his upper bound is not achievable for every positive integer n. For example, if $n = 3$ then Hadamard's upper bound is $3\sqrt{3} \approx 5.2$, but the best that can be achieved is 4.

A *Hadamard matrix* is an $n \times n$ matrix that achieves Hadamard's bound. The *Hadamard conjecture* is that a Hadamard matrix exists whenever n is a multiple of four. I will consider how close to Hadamard's bound we can get when n is *not* the order of a Hadamard matrix, and outline a recent proof that Hadamard's bound is within a constant factor of the best possible, provided n is close (in a sense that will be made precise) to the order of a Hadamard matrix. In particular, if the Hadamard conjecture is true, then the constant factor is at most $(\pi e/2)^{3/2}$. This is joint work with Judy-anne Osborn and Warren Smith.