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# Characterizing Complex Behavior in (Self-Organizing) Multi-Agent Systems

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## Abstract

In this paper, we show our work on characterizing complex behavior in self-organizing (SOMAS) and non-self-organizing (NonSOMAS) multi-agent systems. Through experiments and analysis, we investigate how Self-Organized Criticality (SOC) phenomena arise in SOMAS rather than in NonSOMAS. Furthermore, we compare the order of agent performance in the two types of systems and explain its implications. Finally, we examine how the global patterns evolve in SOMAS as local parameters of individual agents change, and attempt to explain the critical value for the parameters using Shannon's entropy theory.

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# Characterizing Complex Behavior in (Self-Organizing) Multi-Agent Systems

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## Abstract

*In this paper, we show our work on characterizing complex behavior in self-organizing (SOMAS) and non-self-organizing (NonSOMAS) multi-agent systems. Through experiments and analysis, we investigate how Self-Organized Criticality (SOC) phenomena arise in SOMAS rather than in NonSOMAS. Furthermore, we compare the order of agent performance in the two types of systems and explain its implications. Finally, we examine how the global patterns evolve in SOMAS as local parameters of individual agents change, and attempt to explain the critical value for the parameters using Shannon's entropy theory.*

## 1. Introduction

In RoboNBA [11] [12] and RoboCup [7], a player needs to cooperate with its teammates such that its team can perform better. For example, a player tends to pass the ball to a teammate that is in a good position, or tries to run to a position where it is not only ready to attack but also safe to catch the ball. Players exhibiting this cooperative behavior interact with each other and produce aggregated effects on the team.

Generally speaking, there are two types of multi-agent systems: self-organized (SOMAS) and non-self-organized (NonSOMAS). Under certain conditions, the aggregated behavior of multi-agent systems (MAS) cannot be directly understood from the individual agent behavior due to complex interactions among agents, and thus they are considered as SOMAS exhibiting emergent and complex behavior [15][18][23]. The aforementioned cooperative player team is a good example of SOMAS. On the other hand, if the behavior of agents cannot aggregate together, the MAS is considered as non-self-organized.

In order to characterize complex behavior in MAS, we need to evaluate the following properties of MAS:

1. Is there any Self-Organized Criticality (SOC) phenomenon in the system? SOC, as described by Bak [3][4], manifests in a wide range of natural and synthetic systems. Examples of natural systems are living systems [1], national GDP [22], and cities [26]. On the other hand, random boolean networks [5][6], sandpile models [4], and cyberspace [2][13][25] are relevant synthetic systems. Basically, SOC refers to the phenomena that power law distributions appear in natural or synthetic systems. In addition, Darly [8] attempted to use the idea of "emergence" to explain SOC. However, the problem of under what conditions SOC will appear in MAS has generally been overlooked by researchers in the MAS area.
2. We can check how the order of individual agents and MAS evolve. The order of a system refers to how accurately we can predict its future behavior. The order of a system is defined by entropy in statistical physics. The higher the entropy, the lower the order. However, occasionally it is not appropriate to use the entropy to define the order, such as when time series data are present. On the other hand, the second law of thermodynamics states that the entropy of an isolated system always increases, which mean the order of the system always decreases. Our findings should obey this law.
3. Is there any phase transition in the global patterns of MAS as local parameters of individual agents change? In MAS, a phase transition refers to the phenomenon the global properties of MAS change dramatically when local parameters of individual agents change slightly at certain values. These values are the critical values for the MAS. For more details, readers are referred to [9][10][24].

Now the question that remains is how to induce different complex behavior of MAS. First, we need to specify the behavior of individual agents. These agents must be able to aggregate at both individual-level (low-level) and system-level (high-level). Furthermore, the aggregation at the two levels must interact collectively, which means the low-level aggregation can influence the high-level aggregation and the high-level can also have an impact on the low-level [20]. Finally, we should also characterize MAS under different configurations of individual agents, such as different behavior of agents and different interactions among them.

### 1.1. Organization

The paper is organized as follows. In Section 2, we present and explain the mathematical formulation of our MAS. Section 3 defines and discusses the measurements that we adopt. Experiments and discussions are included in Section 4, followed by Section 5, which concludes the paper.

## 2. Formulation

We study two types of MAS to characterize different behavior generated by them. The first type of systems are NonSOMAS and can be viewed as a prototype. The second type, SOMAS, is an extension of NonSOMAS.

### 2.1. Non-Self-Organizing Multi-Agent Systems (NonSOMAS)

In NonSOMAS, there are no critical interactions between agents, and therefore agents can be approximately viewed as independent of each other. Hence, one single agent is able to simulate the whole system. For example, in the computer simulated football match RoboCup, if players (a player can be consider as an agent) do not cooperate with their teammates, one player is representative enough for the whole team.

The formulation in Subsection 2.1 only considers one player, which competes with the external environment. For each clock cycle, if the player beats the external environment, its performance will be increased; otherwise it will be deducted. This is designed according to the spirit "only the fittest will survive" or positive feedback [19], in short. The update function of player performance is defined as:

$$\alpha(t+1) = \begin{cases} \alpha(t) + \beta & Q(\alpha(t)) = 1 \\ \alpha(t) - \beta & Q(\alpha(t)) = 0 \end{cases} \tag{1}$$

where,

- $\alpha(t)$ is the performance of the player at time $t$.
- $Q(x)$ denotes the process that the player competes with the external environment:

$$Q(x) = \begin{cases} 1 & x > rand(\gamma) \\ 0 & x \leq rand(\gamma) \end{cases} \tag{2}$$

- $\beta$ is the step size for $\alpha$.
- $rand(x)$, $x > 0$, generates a random number within the range of [-x, x].

### 2.2. Self-Organizing Multi-Agent Systems (SOMAS)

The formulation in Subsection 2.2 takes into consideration competitive scenarios of MAS. Specifically, it attempts to capture the dynamic process of RoboNBA or RoboCup game: each match is composed of a number of attacks, which themselves consist of a number of clock cycles. At each clock cycle, one player is selected from each of the two teams and competes with each other. Note that we use some approximations, such as constant intervals of attacks and clock cycles.

**2.2.1. The Process** A complete competition process is divided into a number of attacks, and an attack is composed of several clock cycles. The dynamics of SOMAS can be described by a set of variables and update functions (By dynamics, we mean the different behavior of an agent or MAS as time evolves):

- $\alpha_{ij}(t)$ denotes the performance of player $j$ from team $i$ at time $t$, where $i \in \{0, 1\}$ and $j \in \{1, 2, \cdots, 5\}$
- The update function of $\alpha_{ij}(t)$ is defined as:

$$\begin{aligned} \alpha_{ij}(t+s) = {} & F(\alpha_{01}(t+s-1), \cdots, \alpha_{05}(t+s-1), \\ & \alpha_{11}(t+s-1), \cdots, \alpha_{15}(t+s-1)), \\ & where,\ mod(t,m) = 0\ , 1 \leq s < m. \\ \alpha_{ij}(t+m) = {} & G(\alpha_{01}(t+m-1), \cdots, \alpha_{05}(t+m-1), \\ & \alpha_{11}(t+m-1), \cdots, \alpha_{15}(t+m-1), \\ & R_i(t), R_j(t), M_i(t/m), M_j(t/m)), \\ & where,\ mod(t,m) = 0. \end{aligned} \tag{3}$$

where,

- $m$ is the number of clock cycles of an attack.
- $mod(t, m)$ denotes the operation modulus after division.

**2.2.2. Low-Level Aggregation** At each clock cycle, one player is selected from each team and competes with one another. The performance of players is adjusted accordingly in favor of positive feedback. In addition, the results of competitions are recorded. We consider the dynamics of performance as low-level aggregation, since it is defined on players, which are the lowest-level components in SOMAS. The following describe the dynamics of low-level aggregation:

- $F$ updates the performance at all clock cycles except the last one of an attack.

$$F = \begin{cases} \alpha'_{ij} & H(t+s, i) \neq j \\ \alpha'_{ij} + \beta & H(t+s, i) = j \ \&\&\ Y(\alpha'_{ij}, \alpha'_{nz}) = 1 \\ \alpha'_{ij} - \beta & otherwise \end{cases} \tag{4}$$

- $\alpha'_{ij} = \alpha_{ij}(t+s-1)$ and $\alpha'_{nz} = \alpha_{nz}(t+s-1)$.
- $n = V(i), z = H(t+s, n)$.
- $V(i)$ returns the ID of team $i$'s opponent team.

$$V(i) = \begin{cases} 1 & i = 0 \\ 0 & i = 1 \end{cases} \quad (5)$$

- $H(t+s, i)$ selects a player from team $i$ and returns the player ID at time $t+s$. By default, $H(x)$ uses random selection.
- $Y(\alpha_{ij}(t+s-1), \alpha_{nz}(t+s-1))$ denotes the process that player $j$ from team $i$ competes with player $z$ from team $n$, and decides which one wins.

$$Y(\alpha_1, \alpha_2) = \begin{cases} 1 & \alpha_1 + rand(k) > \alpha_2 + rand(k) \\ 0 & otherwise \end{cases} \quad (6)$$

- $k$ denotes the influential power of the external environment.

**2.2.3. High-Level Aggregation** At the last clock cycle of an attack, the competition records and morale of two teams determine the result of the attack. In addition, the morale of the each team is modified accordingly in favor of positive feedback. We consider the dynamics of morale as high-level aggregation, since it is defined on a team, which is a higher hierarchical organization than players in SOMAS. The morale is influenced by the player performance and can change it in turn. The details are defined in Subsection 2.2.4. The following describe the dynamics of the high-level aggregation:

- $M_i(t/m)$ is the morale of team $i$ at attack number $t/m$. $M_i((t+m)/m)$ is updated by function $P$.

$$P = \begin{cases} M_i(t/m) + \delta & O(R_i(t+m), R_n(t+m), \\ & M_i(t/m), M_n(t/m)) = 1, \\ & where\ n = V(i) \\ M_i(t/m) - \delta & otherwise \end{cases} \quad (7)$$

- $O$ determines which team wins at an attack.

$$O = \begin{cases} 1 & R_i(t+m) - R_n(t+m) + \\ & \chi(M_i(t/m) - M_n(t)) > 0, \\ & where\ n = V(i) \\ 0 & otherwise \end{cases} \quad (8)$$

- $R_i(t)$ denotes the number of winning clock cycles of team $i$ at time $t$. It is only valid within an attack interval.

$$R_i(t+s) = \begin{cases} R'_i + 1 & 1 \leq s < m\ \&\&\ Y(\alpha'_{ij}, \alpha'_{nz}) \\ & = 1 \\ R'_i & otherwise \end{cases} \quad (9)$$

where,

- $n = V(i), z = H(t+s, n)$.
- $$R'_i = \begin{cases} 0 & mod(t+s-1, m) = 0 \\ R_i(t+s-1) & mod(t+s-1, m) > 0 \end{cases} \quad (10)$$

**2.2.4. Collective Feedback** As mentioned before, the morale of the each team is changed according to the competition result. The process as defined in $P$ and $O$ refers to how low-level aggregation influences high-level aggregation. On the other hand, one player is selected from each team and its performance is modified accordingly. The process as defined in $G$ illustrates how high-level aggregation influences low-level aggregation:

- $G$ updates the performance at the last clock cycle of an attack.

$$G = \begin{cases} \alpha^2_{ij} & I(t+m, i) \neq j \\ \alpha^2_{ij} + \beta & I(t+m, i) = j\ \&\&\ O(R_i(t+m), \\ & R_n(t+m), M_i(t/m), M_n(t/m)) \\ & = 1,\ where\ n = V(i) \\ \alpha^2_{ij} - \beta & otherwise \end{cases} \quad (11)$$

where,

- $\alpha^2_{ij}$=$\alpha_{ij}(t+m-1)$.
- $I(t+m, i)$ selects a player from team $i$ at time $t+m$ and returns the player ID. It is similar to $F(t+s, i)$. By default, it uses random selection.

Table 1 specifies parameters used in SOMAS.

| | Initial value | Range | Remark |
|---|---|---|---|
| $\alpha$ | 5000 | 0-10000 | performance for a player |
| $M$ | 5.0 | 0-10 | morale for a team |
| $\gamma$ | 10000 | a constant | range of a random number |
| $m$ | 6 | a constant | no. of clock cycles per attack |
| $\beta$ | 1 | a constant | step size for performance |
| $\delta$ | 0.005 | a constant | step size for morale |
| $\chi$ | 0.1 | a constant | an coefficient in $O$ |

**Table 1. Parameters used in SOMAS**

## 3. Measurements

In order to characterize the aforementioned complex behavior, we have designed the following measurements.

- *The order of low-level and high-level aggregation.* The order can be defined by the autocorrelation function: $C(\tau) = lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} c(t_i)c(t_i+\tau)$ for discrete systems. We divide the sum by the number of $N-\tau$ to ensure fairness for each $C(\tau)$. If $C(\tau)$ decreases relatively slowly, $c(t)$ is considered relatively ordered. If $C(\tau)$ decreases rapidly (e.g., an exponential decay), $c(t)$ is considered as less ordered.

- *''Avalanches''*. It refers to the various degree of changes in player performance or team morale. The size of an avalanche is defined as the number of steps of continuous increases or decreases. We call them avalanches because they are similar to those described in [4].
- *Distribution of avalanches*. It can be an exponential distribution, a power law distribution, or even a random one. The type of distribution makes a profound difference.

## 4. Experiments

We have conducted three sets of experiments to study the problems defined in Section 1.

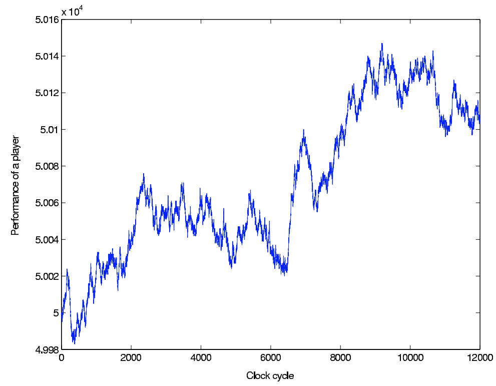
### 4.1. Experiment 1

This experiment is carried out to examine the dynamics generated by NonSOMAS. Figure 1 is a typical output for extensive experiments. Figure 1 (a) illustrates the performance of the player as a function of time. From Figure 1 (b), we can see that the performance avalanche follows an exponential distribution, which indicates that a random variable is dominating the dynamics. Let the fixed probability for an increase in performance be $p$ and then the probability for a corresponding decrease will be $1-p$. So the probability for a $n$ step continuous increase or decrease will be $(1-p)p^n$ or $p(1-p)^n$, respectively, given changes are independent of one another. Note we use 5000 for the initial value of $\alpha$ and it is compared with $rand(10000)$. Therefore, the probabilities to increase or decrease the performance are equally 0.5 and the probabilities for a $n$ step increase or decrease are $0.5^{n+1}$. Consequently, the performance avalanche follows an exponential distribution if the random variable dominates the dynamics and proper initial values are used.

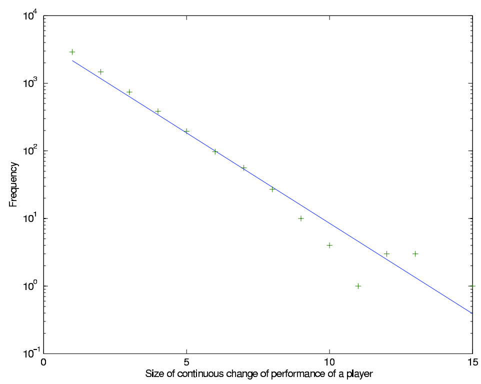
### 4.2. Experiment 2

This experiment is carried out based on SOMAS. Through comparison on Experiment 1 and Experiment 2, we try to have some basic understanding on the Self Organized Criticality phenomena appearing in SOMAS.

Figure 2 is a typical output for extensive experiments. Figure 2 (a) illustrates the performance of a player as a function of time. From Figure 2 (b), we can see that the performance avalanche follows a power law distribution, which is significantly different from the exponential distribution we observed in Experiment 1. This result is quite robust since it appears under different conditions. For example, we use a wide range of $k$ values in $Y(x,y)$ function, and the power law distribution is still there. We are quite confident at the moment the dynamics is not dominated by a random variable otherwise an exponential distribution will appear. What are the underlying mechanisms that generate this power law distribution? We would like to refer to SOC proposed by Bak [3] to explain the underlying mechanisms: under particular conditions, the system organizes by driving itself slowly and eventually it comes to a critical point,

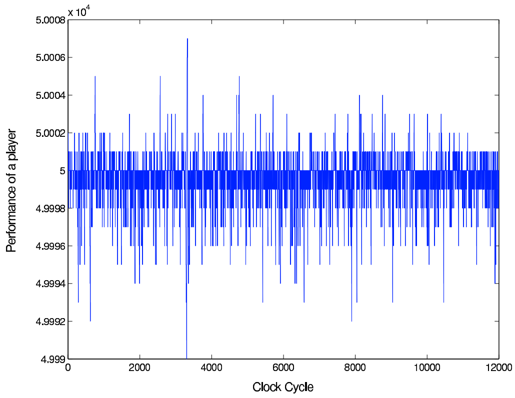



(a) Dynamics of performance of a player in NonSOMAS (x axis: performance , y axis: clock cycle).

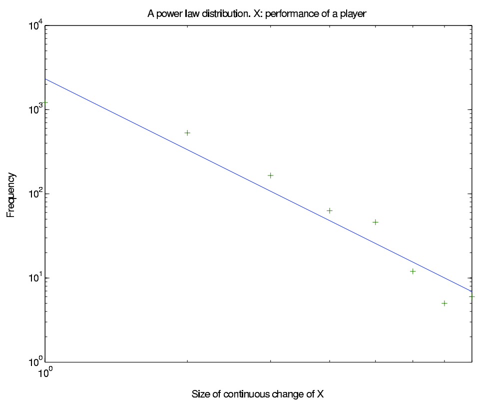



(b) An exponential distribution of performance avalanche (x axis: size of performance avalanche, y axis: frequency).

**Figure 1. Experiment 1: Dynamics and some regularities of player performance in NonSOMAS.**

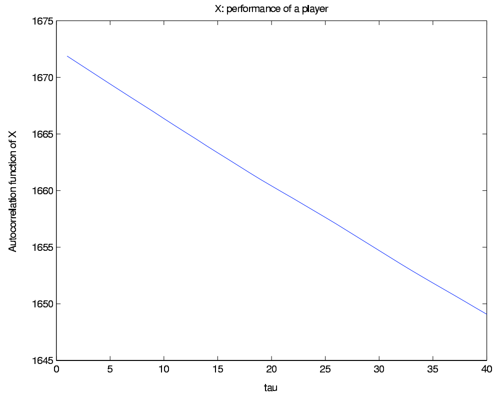



(a) Dynamics of performance of a player in SOMAS (x axis: performance , y axis: clock cycle).

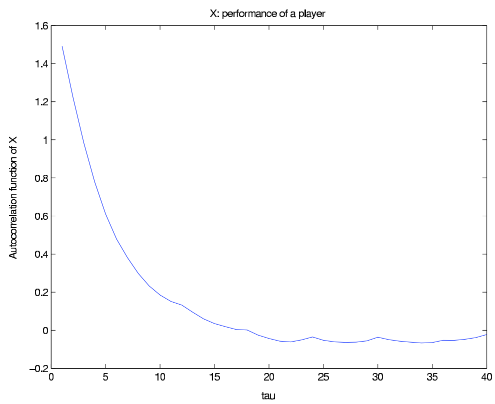



(b) A power law distribution of performance avalanche (x axis: size of performance avalanche, y axis: frequency).

**Figure 2. Experiment 1: Dynamics and some regularities of player performance in SOMAS.**




(a) Autocorrelation as a function of $\tau$ on performance of a player in NonSOMAS (x axis: $\tau$, y axis: autocorrelation function).




(b) Autocorrelation as a function of $\tau$ on performance of a player in SOMAS (x axis: $\tau$, y axis: autocorrelation function).

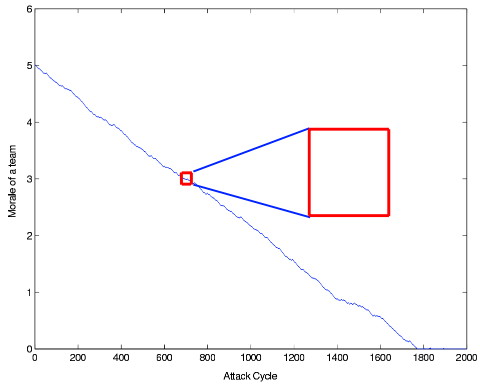
**Figure 3. Comparison on the speed of autocorrelation function decay for player performance as $\tau$ increases.**

where it exhibits complex behavior, such as power law distribution (scale free in space). However, Bak did not point out specific conditions under which SOC would appear.

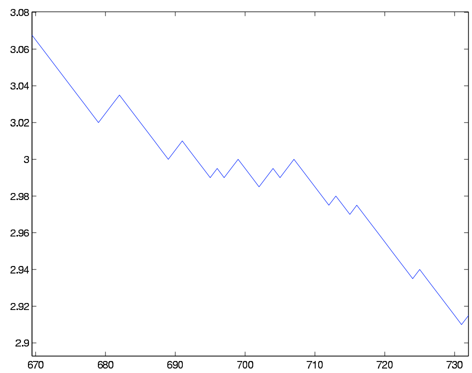
For our general competitive multi-agent model, SOMAS, we consider the SOC phenomena appearing in it is due to the following reasons:

1. Critical interactions between players are important. In NonSOMAS, there is no critical interaction between players and their behavior is relatively ordered and predictable. On the other hand, in SOMAS, there are a total of ten players interacting together. Thus player behavior is relatively harder to predict and thus less ordered. From Figure 3, we can observe the remarkable difference between the order of player performance in the two models. From Figure 3 (a), we can see that the autocorrelation function value is decreasing quite slowly for NonSOMAS. However, in Figure 3 (b), there is an exponential decay as $\tau$ increases, which indicates lower order and predictability for player performance in SOMAS.

2. A higher level organization, team morale ($M$) in SOMAS, is essential. Due to the existence of team morale and its collective interactions with player perfor-

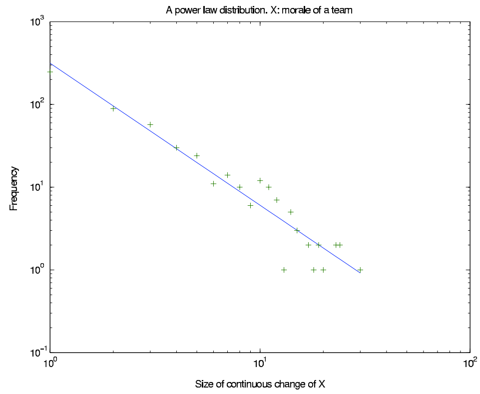



(a) Dynamics of morale of a team (x axis: morale, x unit is 0.005, y axis: attack cycle). The rectangle is enlarged in Figure 4 (b)

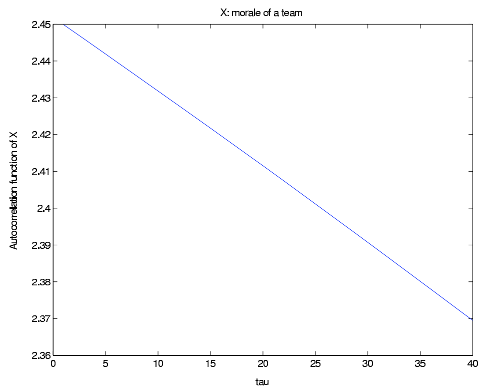


(b) A sub plot of Figure 4 (a) (x axis: morale [670-730], y axis: attack cycle [2.90-3.08]).

**Figure 4. Experiment 2: Morale dynamics of a team in SOMAS.**




(a) A power law distribution of morale avalanche of a team (x axis: size of morale avalanche, y axis: frequency).




(b) Autocorrelation as a function of $\tau$ on team morale in SOMAS (x axis: $\tau$, y axis: autocorrelation function).

**Figure 5. Experiment 2: Regularities of morale of a team in SOMAS.**

mances through functions $G$, $O$, and $P$, we have a collective design of the model rather than more traditional ones, such as top-down or bottom-up design. Thanks to this collective design and backward causation embedded inside it, complex behavior of the model becomes possible [14][17][20]. Figure 4 illustrates a typical dynamics for team morale. From Figure 5 (a), interestingly, we can observe a power law distribution on the team morale avalanche. Furthermore, we can observe a slow decay in Figure 5 (b).

In summary, we can observe SOC phenomena in both low-level (performance) and high-level (team morale) aggregation in SOMAS. Furthermore, thanks to the collective design specified in Subsection 2.2, the SOC phenomena exhibit by decreasing the order on the lower-level aggregation and preserving relatively high order on the higher-level aggregation in the model. This result is consistent with the properties of self-organization process in MAS [16] .

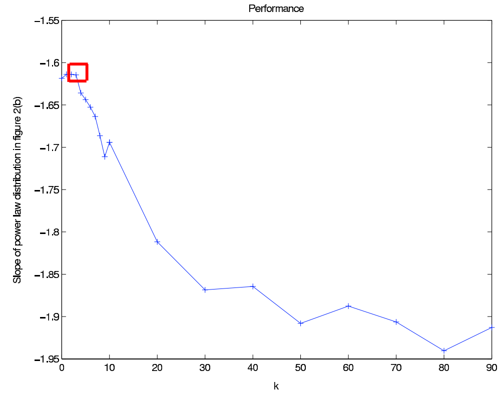
### 4.3. Experiment 3

In this experiment, we try to study the relationship between players' local behavior and global characteristics of SOMAS. Particularly, we try to discover phase transitions embedded in the dynamics produced by the model. Local behavior of players has a broad range of interpretations, but we try to focus on the parameters of equations in the model. In addition, we make sure that these parameters can only have a local influence.

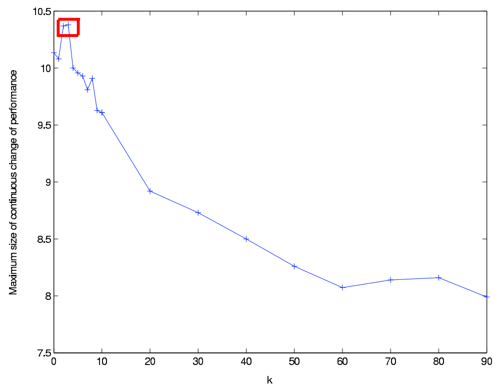
Figure 6 shows how the $k$ parameter in function $Y(\alpha_1, \alpha_2)$ affects the slope of power law distribution plotted in Figure 2 (b) as well as the maximum size of performance avalanche of a player. Similarly, Figure 7 illustrates how the $k$ parameter has an impact on the power law distribution plotted in Figure 4 (b) as well as the maximum size of morale avalanche. For each value of $k$, a hundred data are collected for each point in Figure 6 whereas twenty data are collected for that in Figure 7.

The slope of power law distribution is equivalent to its scaling structure. From Figures 6 (a) and 7 (a), we can observe the slope increases as $k$ increases from zero. The slope peaks as $k$ reaches around three or four. After that the slope decreases dramatically as $k$ further increases. When $k$ reaches 50, the slope begin to fluctuate within a certain range and it can be described as "meta-stable". Furthermore, we can observe the same trends for the maximum size of performance avalanche and morale avalanche in Figures 6 (b) and 7 (b). Therefore, we discover a critical value for $k$, at which both the scaling structure and maximum avalanches of lower-level aggregation (performance) and higher-level aggregation (team morale) reach their peaks, respectively.

When $k$ equals three, the maximum difference produced by two random generators in function $Y(\alpha_1, \alpha_2)$ is six or eight. From Figure 2 (a), we can see for most of time, the performance are within [49995, 50003], and the maximum difference is eight. This range is also valid for the performance differences between other players. So the maximum possible difference between $\alpha_1$ and $\alpha_2$ is around eight. It is amazingly coincidental that the two values in the function $Y(\alpha_1, \alpha_2)$ are so close. From Shannon's entropy theory [21], we know when the two probabilities are equal, the function $Entropy = \sum_i^2 p_i(1 - p_i)$ reaches its maximum. In another word, the entropy (diversity) of a system consisted of two forces reaches its maximum when the two forces are equally strong. We thus conjecture when the difference of $\alpha$ values and random variables are close, the diversity of outcomes they generated reaches its maximum, leading to maximum value of scaling structures and size of avalanches.




(a) Slope of the power law distribution of performance avalanche of a player as a function of k in SOMAS (x axis: k, y axis: slope).

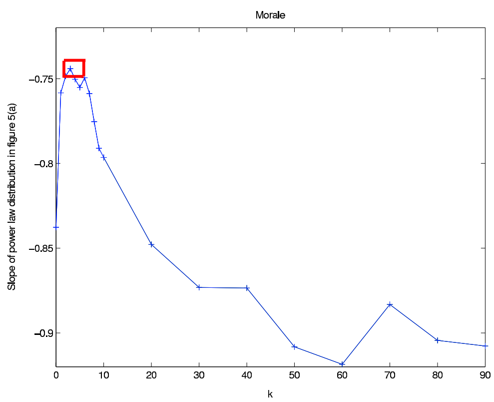



(b) Maximum size of performance avalanche of a player in SOMAS as a function of k (x axis: k, y axis: maximum size).

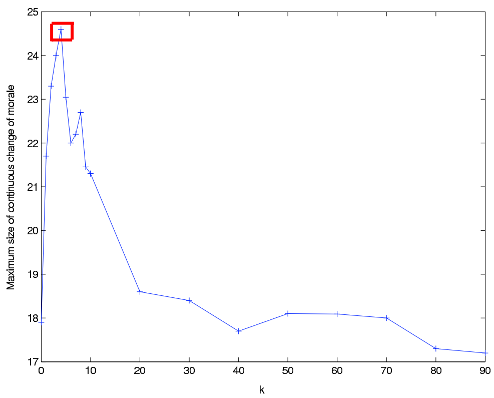
**Figure 6. Experiment 3: Sensitivity of k on some attributes of performance**

## 5. Conclusions

In this paper, we built models for two types of MAS (NonSOMAS and SOMAS) in order to induce and characterize complex behavior in MAS. Based on the two models, we carried out three experiments. Comparing and analyzing the experimental results, we gained some understanding of the Self-Organized Criticality phenomena that occurred in SOMAS. We discovered that the avalanches of player performance and team morale followed a power law distribution. Furthermore, we identified that a parameter $k$ in $Y(\alpha_1, \alpha_2)$ had a great impact on the scaling structures of player performance and team morale as well as maximum size of avalanches. Finally, using Shannon's entropy theory, we provided a conjecture that gives some insight into the phase transitions produced by the critical parameter $k$.




(a) Slope of the power law distribution of morale avalanche of a team as a function of k in SOMAS (x axis: k, y axis: slope).




(b) Maximum size of morale avalanche of a team in SOMAS as a function of k (x axis: k, y axis: maximum size).

**Figure 7. Experiment 3: Sensitivity of k on some attributes of morale.**

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