Nonlinear models of intermittent dynamics

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Based on Vince Darly, Chapter 4, Towards a Theory of Autonomous, Optimizing Agent, PHD thesis, Department of Economics, Harvard University, June 1999

Available at:

http://www.santafe.edu/~vince/pub/dissertation.pdf

Introduction

- Two types of non-linear analysis to the models in Chapter 3:
  - Predictive choice process used by each agent.
    - Complicated dynamics
    - Investigate and prove: chaos and strange attractors
    - Analyze predictors in high dimensionality
  - Statistical physics – to demonstrate different regimes of behaviors

Summary of analysis(1)

- Systems, in which each agent adaptively searches for a good model of its environment, exhibit persistent dynamics, cyclic oscillations between low and high complexity states.
- Two assumptions:
  - Dispersed Interaction: The action of any given agent depends upon:
    - Anticipated actions of other limited agents
    - Aggregate state of these agents
  - Continual Adaptation: behaviors, actions, strategies, and products are revised based on experiences.

Summary of analysis(2)

- Analytical goals: to understand
  - the process which causes the coordination between agents’ strategies
  - The meta-stable equilibrium which leads to complex oscillatory dynamics
- Two approaches:
  - Adaptively rational agents
  - Statistical physics

Basic equations

- Assume we have a market with a single product. Let the price at time *t* be p_t.
- Both supply and demand curves are linear functions of the price:
- Supply: $S(p)=bp$
- Demand: $D(p)=A-Bp$
- p^* is the unique steady state equilibrium: $S(p^*)=D(p^*)$

An ARED model

Consider the state of the system in its transition from date *t-1* to date *t:*

- Using model *j*, at time *t-1*, quantity $S_{j,t} = bH_{j,t} = bp^{e}_{j,t}$

$p^{e}_{j,t}$ means the expected price of model *j* at time *t*

- Hence the total supply is : $S = \sum_j n_{j,t-1} S_{j,t}$
- The utility of each model $U_{j,t}(p_t, S_{j,t})$ can not be directly observed
- Agents observed utilities $\tilde{U}_j = U_j + \tilde{\epsilon}_j$. The probability using model *j* is: $$\mathrm{Prob}(j) = \frac{\exp \beta U_{j,t}}{\sum_j \exp \beta U_{j,t}}$$
- The fraction of agents using model *j* is: $$n_{j,t} = \frac{\exp \beta U_{j,t}}{\sum_j \exp \beta U_{j,t}}$$

Modifications to the ARED model

- To observe the details of the dynamical process, the ARED model is extended to include a system inertia. That is, at each date *t*, each agent makes an adjustment to its model with probability: λ , so with 1- λ , agents retain its old model. So the fraction of agents are: $n_{j,t} = (1-\lambda)n_{j,t-1} + \lambda\frac{\exp \beta U_j}{Z},$
where *Z* is a usual normalizing term.
- Some analysis:
  - Small λ →system has high inertia: small numbers of agents may change their model
  - If $\lambda \approx 1$, system if fluid, changes in population occur instantaneously.

Simplify the model

- We consider a model with 2 predictors: H_1 and H_2. Let $m_t = n_{1,t} - n_{2,t}$, and assume:
  - H_1 is a more sophisticated predictor, and charge costs $C \geqslant 0$.
  - H_2 is free.
- So we have:

$$p_{t+1} = -\frac{b}{2B}\{(1+m_t)H_1(p_t,\ldots)+(1-m_t)H_2(p_t,\ldots)\}$$

Simple short memory predictors

- Let H_1 =p*=0, H_2 =p_t. That means,some agents know that the equilibrium price is 0, but they need to pay a cost C, the others predict that today's price will be the same as yesterday's price. So the update map is :

$$p_{t+1} = f(p_t, m_t) = -\frac{b}{2B}(1 - m_t)\, p_t$$

$$m_{t+1} = g_\beta(p_t, m_t) = (1-\lambda)m_t + \lambda \times \tanh \frac{\beta}{2}\left\{ \left(\frac{b}{B}[1-m_t] + 1\right)p_t^2 - C \right\}$$

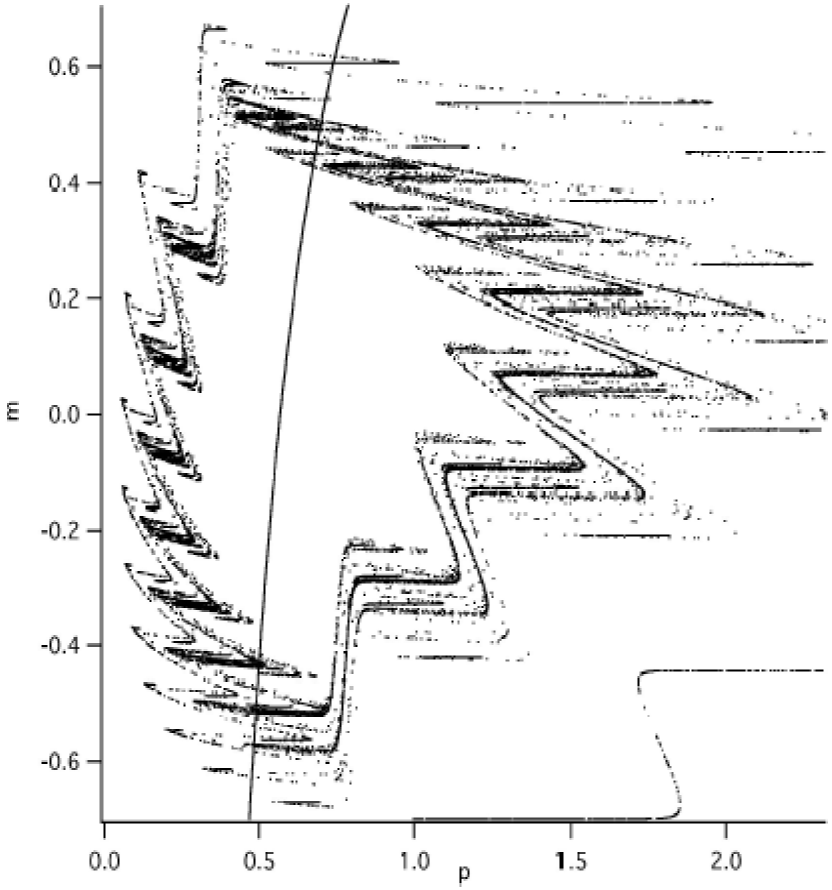
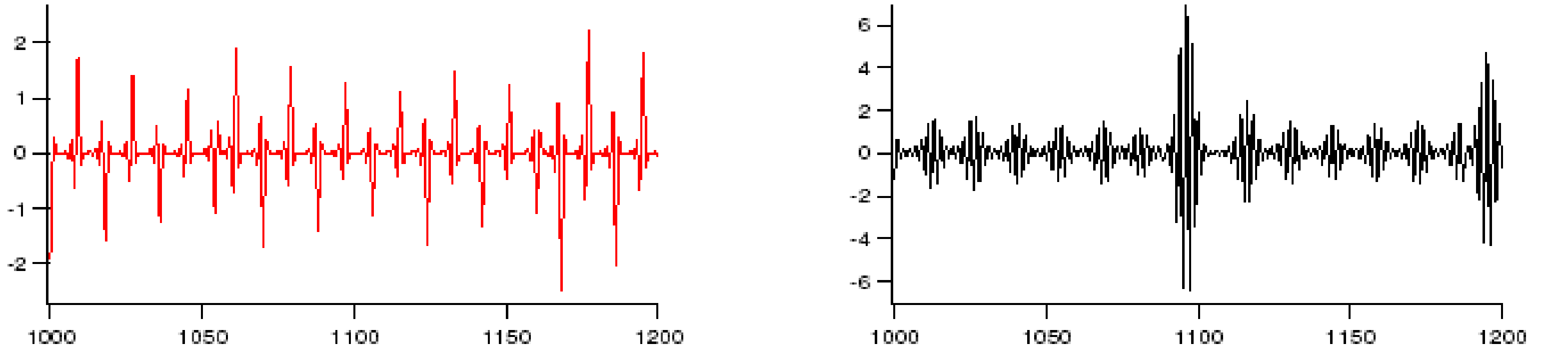


Figure 4.1: The attracting set, $\lambda = 0.15$. The horizontal axis is price, and the vertical axis the population fraction difference m, defined in the text. Since $m = \pm 1$ are the only states in which all agents use the same predictor, the attracting set represents a complex, fluctuating balance between the two predictors, with neither having an overwhelming advantage.



(a) $\lambda = 0.95$

(b) $\lambda = 0.15$

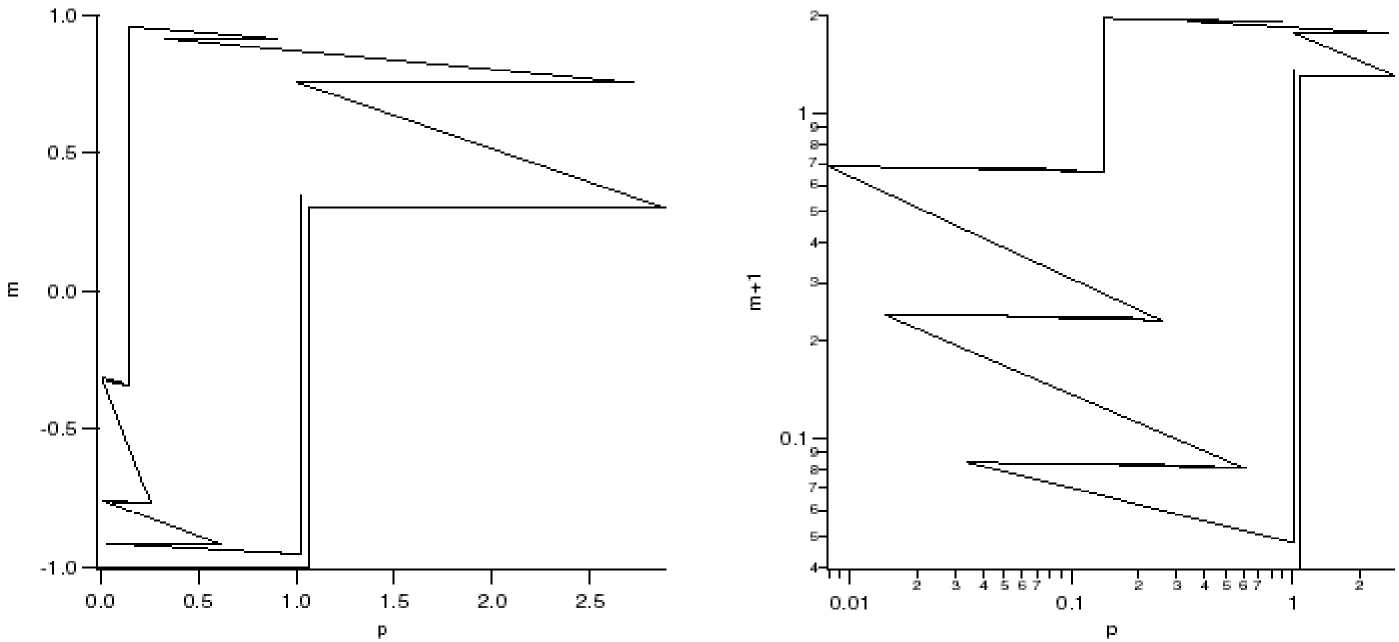
Figure 4.2: Price timeseries. The equilibrium price has been arbitrarily fixed at zero.

Why use these simple models?

- Many properties of this simple model appear to be followed by more complicate predictors
- It exhibit the same dynamics as the more reasonable pairs of predictors we examine later, and the reasons are the same
- Strong empirical similarities dynamics and mechanisms which drive the dynamics.
- Root idea:
  - If all agents use H_1, then this drives the system to equilibrium, then H_2 is effective too;
  - So agents switch to cheaper model H_2.
  - Then system becomes unstable and fluctuate, which will encourage agents switch back to H_1
  - This analysis provides analytical insights into the characteristics of the predictors.

Some results for this model(1)

- The shape of unstable manifolds for $\beta = \infty$,which means 'unboundedly rational'.

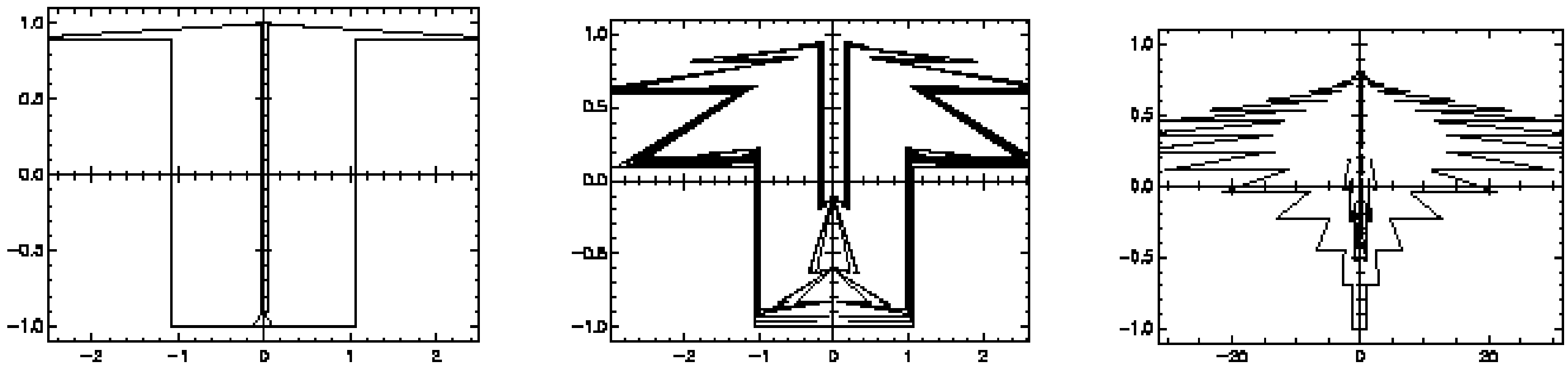


(a) The unstable manifold for $\lambda = 0.65, \beta = \infty$, where we have included the discontinuous line segments (which are part of the manifold for $\beta < \infty$)

(b) Using a logarithmic scale to show more detail of the return path; each denser region contains a switchback in the path

Some results of this model(2)

- There exists significant parameter ranges, with $\lambda c < \lambda < 1$, for which a horseshoe map exists.



(a) $\lambda = 0.95$ (b) $\lambda = 0.55$ (c) $\lambda = 0.15$

Figure 4.4: The unstable manifold of the unique equilibrium point for varying λ, and $\beta = \infty$. Other parameters are: $b = 1.35$, $B = 0.5$, $C = 1$. The horizontal axis representes the price p, and the vertical axis the model proportion $m \in [-1, 1]$. These trajectories are calculated exactly using the results of Theorem 1.

To be continued

- This presentation is just a part of Chapter 4. Further analysis and more complicate models will be introduced.
- What we should learn is the analytical method, from simple to complicate, from ideal to realistic.