Natural Rationality

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Based on Vince Darly, Chapter 3, Towards a Theory of Autonomous, Optimizing Agent, PHD thesis, Department of Economics, Harvard University, June 1999

available at:
http://www.santafe.edu/~vince/pub/dissertation.pdf

A Predictive System

Definition 3.3.1 *A predictive system* $\mathcal{P}$ *is a sextuplet* $\langle N, L, A, B, M, u \rangle$*, where:*

N *is the number of agents in the system,*

L *is a regular lattice with exactly* N *sites,*

$A = \{a_1, a_2, \ldots, a_N\}$ *is the set of agents,*

B *is a Euclidean space of possible behaviours,*

M *is the space of possible predictive models,*

$u : B \times B^{|N|} \rightarrow \mathbb{R}$ *is the utility.*

Behavior is like strategy: [1,1000]

Different ways to predict, based on T and c

Calculation based on its own and neighbors' behaviors

Dynamics Update Rule

Prediction$(i, \mathcal{P})$ at time t:

i. Calculate private predictions $b^j_{t+1,e} \in B$ giving the expected behaviour of all other agents $(j \neq i)$ at time $t+1$.

Predict others' behavior

ii. Find $b^* = \arg\max_{b \in B} u(b, \{b^j_{t+1,e}\})$, agent a_i's predicted optimal response.

Find the optimized behavior

iii. Carry out action b^*.

iv. Observe actual behaviours b^j_{t+1} and calculate agent a_i's utility $u^* = u(b^*, \{b^j_{t+1}\})$.

Calculate the utility based on other's observed behaviors

v. If $\exists \mu'_i$ with $T' = T \pm 1$ or $c' = c \pm 1$ s.t. $u(b^*_{\mu'_i}, \{b^j_{t+1}\}) > u^*$ then pick the *best* such new model μ'_i. This is the model update rule.

Update model by slightly adjust T and c

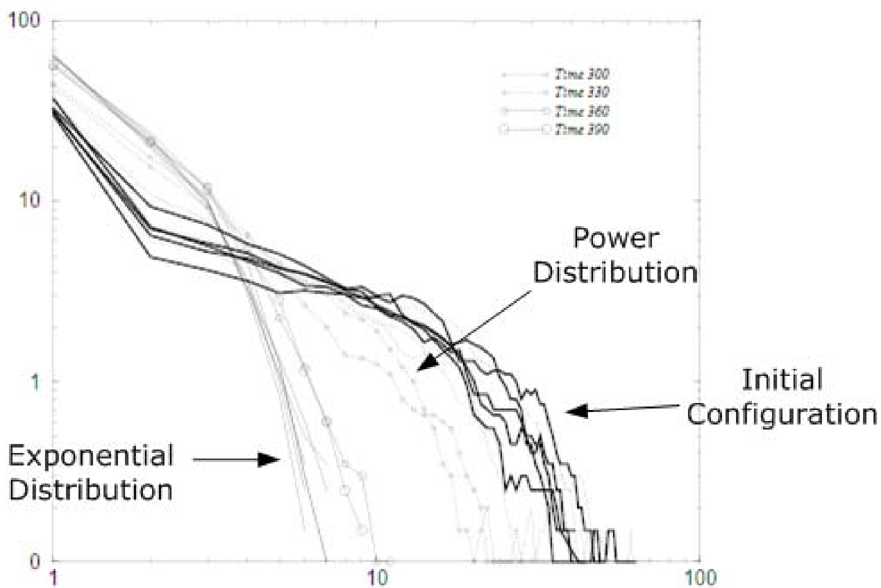
Methodology of observation

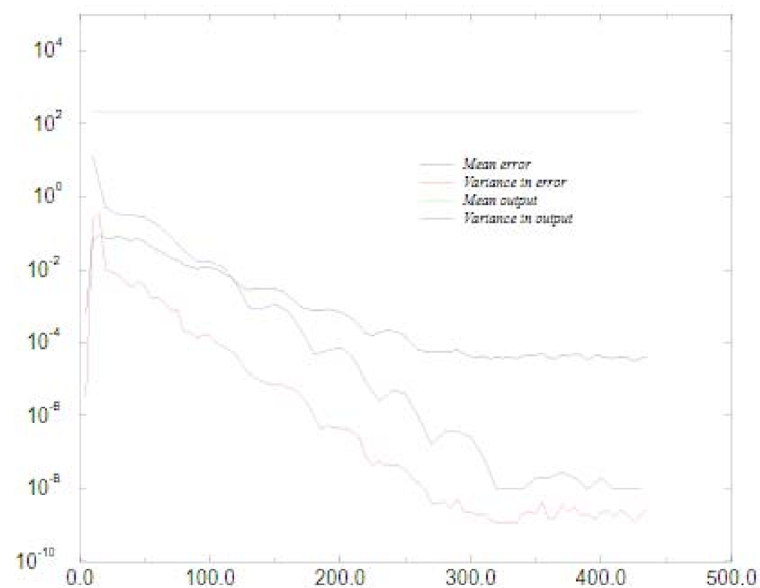
- Forward predictive scenario
  - Natural, obvious predictive method
  - Predict neighbor's action and determine its own behavior using utility
  - Predictions are recorded as history.
- Stabilized scenario
  - Adjust their models using predictions
  - Predictions are then forgotten
  - Iterative refinements

History and Complexity Issues

- Short histories – Not enough data to specify the model accurately
- Long histories – Neighbors may already change their models
- Too simple (low complexity) will not be able to predict time-series
- Too complex (high complexity) may not have enough data to specify the model, also prone to over-fitting

Stabilized Scenario

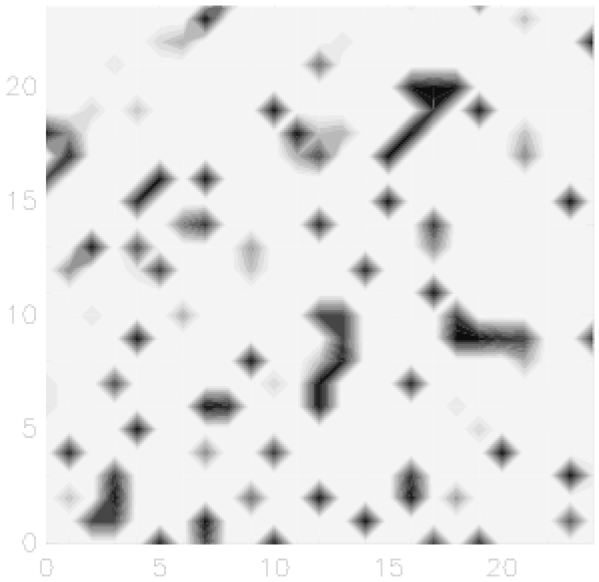




- X: Disturbance size
- Y: Number of Occurrences

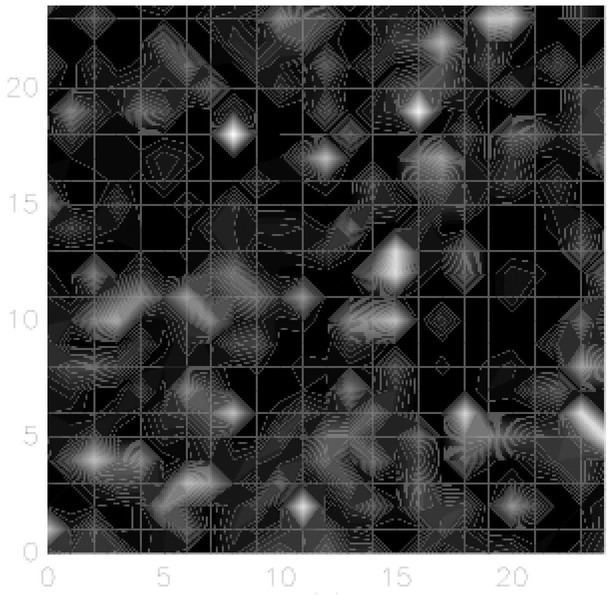
Errors are decreasing to the Rounding errors – limit of errors

Stabilized Scenario continued



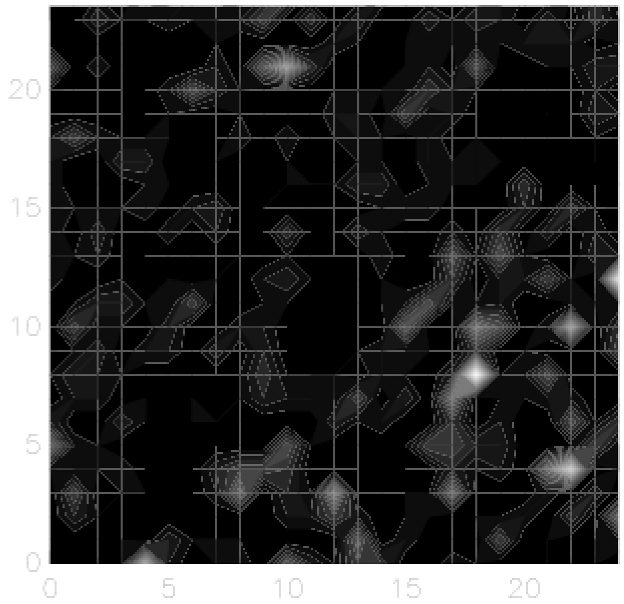
- First transitory phase – gradually removes randomly generated past
- The behaviors of agents easily changed – coherence has not been established among neighbors
- Its duration depends on the degree of correlation between that past and natural dynamics of the system
- In this case, around 50 steps are needed to settle down

Stabilized Scenario continued



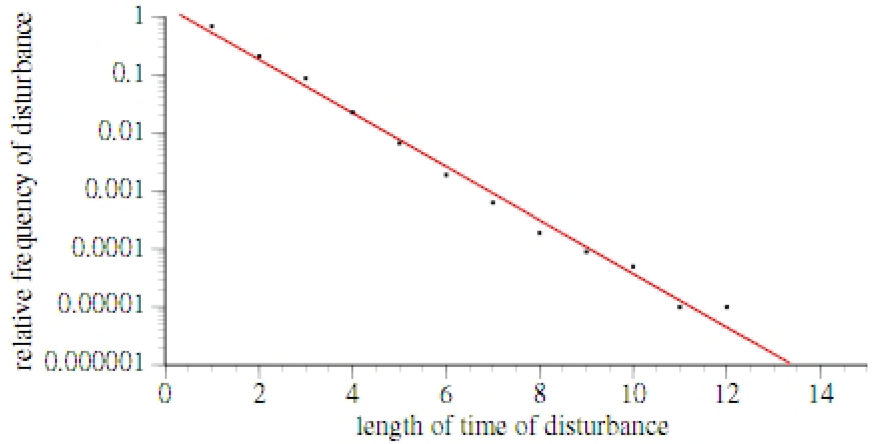
- After the presence of past disappeared, the system ran into relatively fixed scenario (regime 1), although occasional consecutive adjustment can reach to one hundred steps
- High coherence between neighbor's outputs and widely varying spectrum of adjustment lengths
- Spatially the adjustment lengths are clustered together
- Temporally a power-law distribution in the size of adjustments (avalanches)

Stabilized Scenario continued



- The dynamics become largely random
- Spatial coherence disappeared
- Temporal characteristics change from power law distribution to exponential distribution
- Qualitative difference between the two distribution – in later one large avalanches virtually do not occur
- Probability of change at a single step can be calculated: 0.323+/- 0.005

Stabilized Scenario continued



- Probability is calculated via this figure
- Why self-organized → random? The output gradually settle down, and variations reach the limit of rounding errors.
- Rounding error contribute more than predictive errors