Natural Rationality

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Based on Vince Darly, Chapter 3, Towards a Theory of Autonomous, Optimizing Agent, PHD thesis, Department of Economics, Harvard University, June 1999

available at:
http://www.santafe.edu/~vince/pub/dissertation.pdf

Background

- Large number of agents interact locally
- They use predictive models, which include history and complexity
- Agents predict the behaviors of each other and choose the behavior to optimize the utility
- Agents can change their predictive models according to certain rules

Assumptions

- Agents have some history to memorize
- Agents have some complexity to predict time series
- Agents can know the previous history of behaviors of its neighbors
- However, agents do not know the models(mechanisms) of its neighbors

Nonlinearity

A linear model of update function for a single agent

$$p_t = a + b(\alpha_0 + \sum_{i=1}^{c} \alpha_i p_{t-i}) + u_t$$

A linear model of minimization

$$\alpha = \arg\min \frac{1}{N} \sum_{i=1}^{T} (p_{t-i} - \alpha_0 - \sum_{j=1}^{c} \alpha_j p_{t-i-j})^2$$

A nonlinear model of update function

$$p_t = F(p_{t-1}, \ldots, p_{t-T})$$

A Predictive System

Definition 3.3.1 *A predictive system $\mathcal{P}$ is a sextuplet $\langle N, L, A, B, M, u \rangle$, where:*

N *is the number of agents in the system,*

L *is a regular lattice with exactly N sites,*

$A = \{a_1, a_2, \ldots, a_N\}$ *is the set of agents,*

B *is a Euclidean space of possible behaviours,*

M *is the space of possible predictive models,*

$u : B \times B^{|\mathcal{N}|} \to \mathbb{R}$ *is the utility.*

Behavior is like strategy: [1,1000]

Different ways to predict, based on T and c

Calculation based on its own and neighbors' behaviors

Dynamics Update Rule

Prediction$(i, \mathcal{P})$ at time t:

i. Calculate private predictions $b^j_{t+1,e} \in B$ giving the expected behaviour of all other agents $(j \neq i)$ at time $t+1$.

ii. Find $b^* = \arg\max_{b \in B} u(b, \{b^j_{t+1,e}\})$, agent a_i's predicted optimal response.

iii. Carry out action b^*.

iv. Observe actual behaviours b^j_{t+1} and calculate agent a_i's utility $u^* = u(b^*, \{b^j_{t+1}\})$.

v. If $\exists \mu'_i$ with $T' = T \pm 1$ or $c' = c \pm 1$ s.t. $u(b^*_{\mu'_i}, \{b^j_{t+1}\}) > u^*$ then pick the *best* such new model μ'_i. This is the model update rule.

Predict others' behavior

Find the optimized behavior

Calculate the utility based on other's observed behaviors

Update model by slightly adjust T and c

Model Details

Predictive Model

$$\alpha^* = (\alpha_0^*, \alpha_1^*, ..., \alpha_c^*),$$

Linear Recurrence Relation

$$b_{t+1,e} = \alpha_o^* + \sum_{t'=1}^{c} \alpha_{t'}^* b_{t+1-t'}$$

How to calculate alpha* ? Minimize least square errors over past history

$$error_{j,t}^*(T,c) = \sum_{\tau=0}^{T-c-1} \{b_{t-\tau}^j - \alpha_0^* - \sum_{t'=1}^{c} \alpha_{t'}^* b_{t-t'-\tau}^j\}^2$$

Model Details continued

A more detailed way to predict

$$b_{i,t+1}^{pred} = \alpha_0^* + \sum_{j \in N_i} \sum_{t'=1}^{c} \alpha_{j,t'}^* b_{j,t+1-t'}$$

Utility functions

Coordination

$$u = -|b - \frac{1}{N}\sum_j b_j|$$

Substitution

$$b_{br} = 1000 - \sum_j b_j$$

Coordination with preference

$$b_{br} = \lambda D + (1-\lambda)\frac{1}{N}\sum_j b_j$$

Comments

- Locality in the paper means:
  - Agents use a short range of history
  - Certain level of complexity is embedded
  - Utility is evaluation on one step
  - Predictive model update locally

Comments continued

- Some issues with the paper
  - Prediction model of neighbors is not explicitly defined
  - The problem of my model of your model of my model …
  - Time series is the central theme
  - Utility based on history?

Next Presentation

- Methodology of observation
- Observation of the stabilized scenario
- Observation of the forward predictive scenario