The Self-Organization Criticality Algorithm

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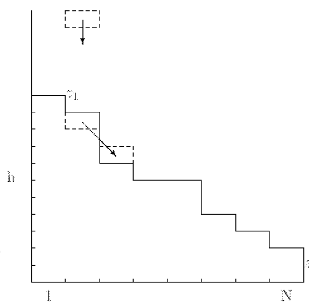
Based on: Vince Darley, Towards a Theory of Autonomous, Optimising Agents, PhD thesis, Department of Economics, Harvard University, June 1999, available at: http://www.santafe.edu/~vince/pub/dissertation.pdf

Self-Organized Criticality

- Dynamical systems:
  - with temporal and spatial degrees of freedom
  - Two commonly observed phenomena in nature: *1/f noise* and *fractal structure*
  - Common feature: Self-similarity in time and space
  - Examples: the luminosity of stars, the dynamics of turbulent flow, the shape of coastline, river erosion pattern
- Self-organized criticality
  - Proposed by Bak et al.
  - To explain the above two phenomena

The Sand-Pile Model

- A cellular automaton developed by Bak et al.
- The one-dimensional model:
  - h_i : the height of sand at site *i*
  - $z_i = h_i - h_{i+1}$: the slope of site *i*
  - When dropping a grain of sand at site *i*:
    - $z_i \rightarrow z_i + 1$
    - $z_{i-1} \rightarrow z_{i-1} - 1$
  - Sand grains are randomly dropped
  - When slope z_i is greater than a certain ***critical value*** z_c , an avalanche takes place:
    - $z_i \rightarrow z_i - 2$
    - $z_{i\pm1} \rightarrow z_{i\pm1} + 1$
  - A single stable attracting state: $\forall i, z_i = z_c$



The SOC-Optimizer

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SOC-OPTIMISER(n, Σ)
  Pick a random solution σ ∈ Σ
  f_σ = FITNESS(σ)
  E(1) ... E(n) ← E_0
  repeat
      i ← RAND(n)
      E(i) ← E(i) + δE
      [σ', δ] ← IMPROVEMENT(σ, i)
      if δ > E_c − E(i)
        then σ ← σ'
             f_σ ← f_σ + δ
             for each j in N_i(σ)
             do
                E(j) ← E(j) + E(i)/|N_i(σ)|
             E(i) ← 0
  until No time remains
  return σ, f_σ
```

The energy of *n* pieces
Analogy: the height of *n* sand sites

Add energy on a randomly selected piece *i*
Analogy: Drop a grain of sand on a random site *i*

Energy of piece I flows to its neighboring pieces
Analogy: Avalanche takes places at site *i*, sand grains fall to neighboring sites in N_i

Solution space, that can be decomposed to *n* pieces
Analogy: *n* pieces → *n* sand sites

A unit amount of energy
Analogy: a grain of sand

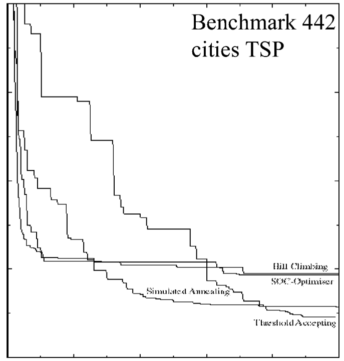
A critical energy state?

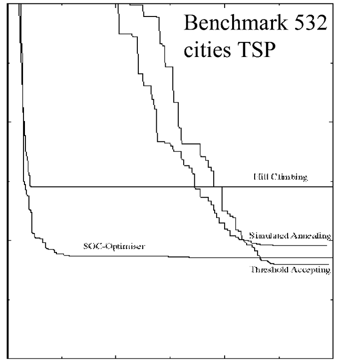
The SOC-Optimizer (Cont.)

- IMPROVEMENT $\Sigma \times N \rightarrow \Sigma \times R$:
  - $\sigma' = \text{PERTURBATION}(\sigma, i)$
  - $\text{IMPROVEMENT}(\sigma, I) = [\sigma', \text{FITNESS}(\sigma') - \text{FITNESS}(\sigma) \mid \sigma]$
- A perturbation operator is local on a problem space Σ if it satisfies the following property:
  - $\forall \sigma \in \Sigma, \forall \sigma' \in \text{PERTURBATION}(\sigma)$,
  - $\text{Computation}\{\text{FITNESS}(\sigma') \mid \sigma, \text{FITNESS}(\sigma)\} \ll \text{Computation}\{\text{FITNESS}(\sigma')\}$

  or equivalently
  - $\forall \sigma \in \Sigma, \forall \sigma' \in \text{PERTURBATION}(\sigma)$,
  - $\text{Computation}\{\text{FITNESS}(\sigma') - \text{FITNESS}(\sigma) \mid \sigma\} \ll \text{Computation}\{\text{FITNESS}(\sigma') - \text{FITNESS}(\sigma)\}$

Optimization Results



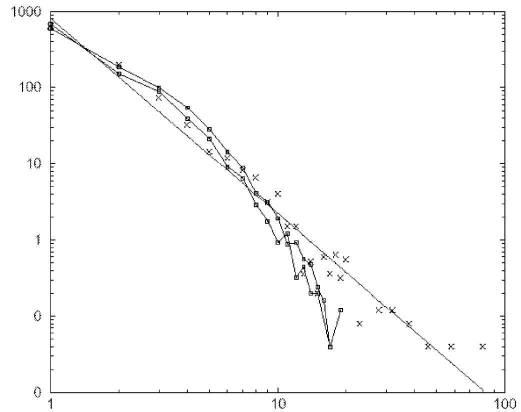


- The SOC-Optimizer performs competitively, but is not better than SA or TA on the test problems
- The SOC-Optimizer less depends on good parameter setting than the others. It finally achieves a slightly poorer solution in general
- The SOC-Optimizer can rapidly find a good, not optimal, solution

Optimization Results (Cont.)

- SOC, SA, TA, and HC improve in fitness rapidly at the start, with hill-climbing in fact the slowest of the four (???)
- Even in the early stage of optimization, it is necessary to avoid getting trapped in poor areas of the search space

Optimization Results (Cont.)



- Avalanche length distributions