Agent Network Topology and Complexity

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ABSTRACT

In this paper, we examine two multi-agent based representations of SATs and further experimentally study the topologies of resulting agent networks. We show that different representations will make agent networks manifest different topologies. In one presentation, the resulting agent network show obviously small-world topologies. Generally speaking, a small-world topology will computationally harden a search process. Therefore, we propose a guiding design principle that to solve a search problem by a multi-agent system, it should avoid having small-worlds among agents and it should maintain balanced intra- and inter- agent computational complexity.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Experimentation, Measurement

Keywords

Agent network, Small-world, Computational complexity

1. INTRODUCTION

Small-world phenomena are widely observed in many systems, ranging from natural systems (e.g., food webs [4]) to man-made systems (e.g., WWW [1]). When using a multi-agent system, such as MASSAT [2], to solve a SAT problem, it will implicitly or explicitly form an agent network. In this paper, we will address the following questions: (1) Unlike those in MASSAT, if agents in a multi-agent system are used to represent clauses in a SAT, what topology can be observed in the resulting agent network? (2) Does an agent network formed in MASSAT show a small-world topology? (3) If the topology of an agent network varies according to specific problem representations, how does it reflect the computational complexity of solving the given SAT problem by the corresponding multi-agent system?

2. TOPOLOGY OF AN AGENT NETWORK

When employing a multi-agent system to solve problems, it normally requires interactions among agents. Here, 'interactions' should be understood in a broad sense. They can be cooperations, competitions or constraints among agents. Because of the interactions, agents in a multi-agent system implicitly or explicitly form a network where vertices denotes agents and edges denotes interactions among agents. Using a multi-agent system to solve a SAT, different representations exist. In the following, we will study two typical distributed agent representations: (1) agents represent clauses and (2) agents represent variables.

2.1 Clause-Based Representation

In this subsection, we will examine the multi-agent representation of a SAT problem where agents represent clauses. In this representation, the agent should make at least one literal true so as to satisfy its clause. Because a variable can appear in multiple clauses simultaneously, the agents that have common variables should cooperately assign these variables with compatible values so as to satisfy their respective clauses. In this situation, an agent acts as a vertex of an agent network. If two agents have a common variable, there will be an edge between the corresponding two vertices.

Based on the above representational encoding scheme, we conducted some experiments on some benchmark SAT problems downloaded from SATLIB: Uniform-3-SAT problems and Flat Graph Coloring problems. As two important properties for characterizing graphs, we calculated the average *characteristic path length*, L_G, and *clustering coefficient*, C_G, of each testset. In general, L_G represents a global property of graph G, which indicates the connectivity of G. On the contrary, C_G shows a local property, which reflects the average connectivity of cliques in graph G [6]. Before further descriptions, let us briefly introduce the notion of *small-world*.

Milgram first proposed the notion of *small-world* [3]. Later, Watts and Strogatz formulated it based on *characteristic path length* and *clustering coefficient* [6]. They defined that graph G has a small-world topology iff: $L_G \approx L_r$ and $C_G \gg C_r$, where L_r and C_r are the average characteristic path length and clustering coefficient of random graphs with the same size as G. This definition of a small-world is qualitative. Walsh further provides a quantitative measure-

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ment, called *proximity ratio* μ [5]: $\mu = (C_G \cdot L_r)/(L_G \cdot C_r)$. A small-world topology requires $\mu \gg 1$. The larger the μ, the more 'small-worldy' the graph (i.e., the graph has more clusters).

To test if there are small-world topologies, in our experiments we calculated the average L_G *and* C_G *as well as* μ *for all testsets. For each instance in a testset, we generated 10 random graphs with the same size as G and calculate* C_r *and* L_r. *The results show that in all testsets,* $L_G \approx L_r$, $C_G \gg C_r$, *and* $\mu \gg 2$. *It means with this representation, the agent networks show small-world topoloties.*

2.2 Variable-Based Representation

In this subsection, we will study: What is the topology of an agent network formed in MASSAT? In SATs, in order to make a clause satisfied, the related variables should be assigned compatible values to guarantee at least one literal is ture. In this sense, there are constraints among variables. Since MASSAT represents variables with agents, the constraints among variables are implicitly transferred to constraints among agents. In order to satisfy a constraint (i.e., a clause), agents that represent variables in the constraint will restrain each other. If we assume each agent represents only one variable, the agent network can be formed as follows: A vertex denotes an agent. There is an edge between two agents if and only if the corresponding two variables appear in a certain clause simultaneously.

Based on the above encoding scheme, we conducted the same experiments as those in previous subsection. Our experimental results show that in all testsets of Uniform-3-SAT, $L_G \approx L_r$, $C_G \approx C_r$, *and* $1.1 < \mu < 1.3$. *This indicates there is no small-world topology. In all the testsets of Flat Graph Coloring,* $L_G \not\approx L_r$, $C_G \not\approx C_r$, *and* μ *is around* 1.6, *i.e., there is no small-world topology, either. Thus, we can assert that with this representation, there is no small-world topology in agent networks.*

In the above encoding, each agent represents only one variable. We can also use one agent to represent several variables. We have experimentally proven that in this case, μ is still less than 2.0.

3. DISCUSSIONS

3.1 Complexities in Different Representations

Given a SAT problem, different representations can lead to different agent networks. In our mentioned two cases, the first one shows small-world topologies in its resulting agent networks. The second one, i.e., MASSAT, does not generate any small-world topology.

As a measurement of computational complexity, Walsh has studied the relationship between topology, in particular, a small-world topology, and complexity. In [5], Walsh empirically proved that a small-world topology can computationally harden the computational complexity of a search algorithm that involves certain heuristics. We have also experimentally validated this assertion based on the previous two representations of SAT problems. Using the same SAT problems, we examined a clause-based representation as opposed to the variable-based representation used in MASSAT. Our experimental results suggest that in such a problem representation, it is normally hard (in term of flips) to solve a problem, while it is relatively easier to solve with MASSAT.

3.2 Balanced Complexities in Intra- and Inter-Agent Computations

In MASSAT, an agent can represent one or more variables. In both cases, the obtained agent networks do not have any small-world topology. But, which case is better? Experiments have suggested that as the number of variables represented by an agent in MASSAT increases, the created agent networks will become less 'small-worldy'. The networks become smaller and denser. As an extreme case, one agent can represent all variables. In this situation, a network collapses to an isolate vertex. Does this mean the best situation? Of course not. Obviosly, if an agent represents multiple variables, the agent should assign values to all its variables. Because the variables of an agent are not independent, as the number of variables represented by an agent increases, the intra-agent computational complexity to assign values to its variables becomes greater. Therefore, a good design should balance intra- and inter- agent computational complexities to achieve the lowest possible total computational cost.

3.3 A Guiding Principle

Based on the above discussions, we can arrive at a guiding principle for designing a multi-agent based approach to solving a search problem: (1) It should avoid involving a small-world topology in a resulting agent network; (2) It should maintain balanced intra- and inter- agent computational complexities so as to achieve the lowest total computational cost.

4. SUMMARY

The originalities of this paper are (1) proposing the notion of agent network and (2) addressing the relationship between the topology of the resulting agent network and the computational complexity of the corresponding multi-agent system in solving a given search problem. Specifically, we addressed two agent-based representations of satisfiability problems. We observed that different representations can generate agent networks with different topologies. Particularly, in the clause-based representation, the resulting agent network shows a small-world topology. Because a small-world topology will computationally harden a search process, we generalized a guiding principle for general agent-based problem solving, that is, (1) avoiding involve a small-world topology and (2) maintain balanced inter- and intra-agent computational complexities.

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