More sophisticated predictors

- Examine 2 predictors:
  - Hypothetical perfect predictor: $H_7(p_t,\ldots)=p_{t+1}$
  - Static predictor: $H_2(p_t,\ldots)=p_t$
- Why?
  - In some cases, we believe a neoclassical perfectly rational agent could predict perfectly, we need to observe the dynamics of the ARED model.
  - We have examined a simple extreme case, if more complex and adaptive ones exhibit chaotic dynamics, and then those intermediate, more realistic cases follow many of the properties of the two extremes, then we can prove these cases also exhibits chaotic dynamics.

Analysis

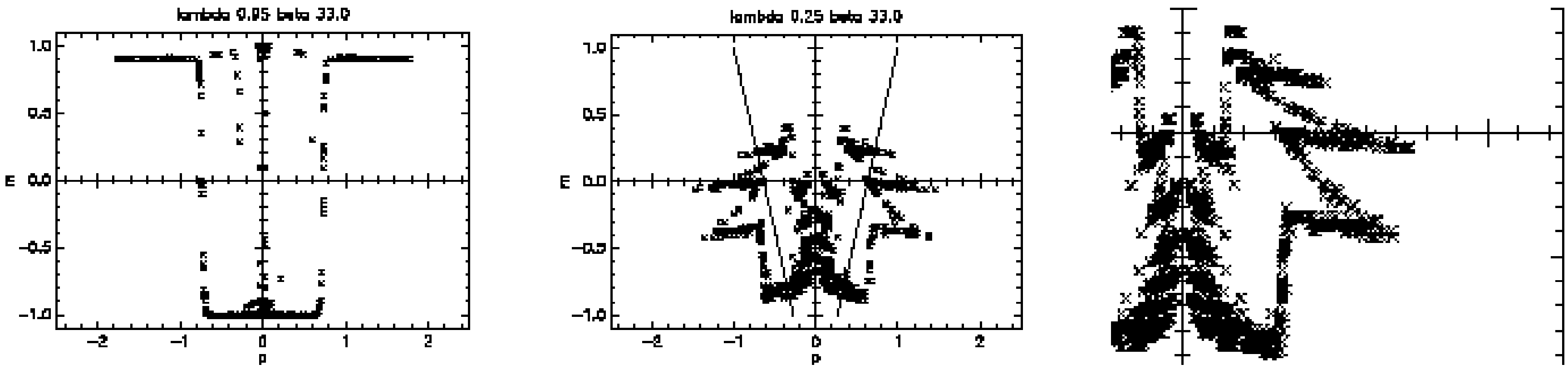
- The equations of predicted price and the population file are:

$$p_{t+1} = f(p_t, m_t) = -p_t \left(\frac{b(1-m_t)}{2B + b(1+m_t)} \right) \quad (4.33)$$

$$m_{t+1} = g_\beta(p_t, m_t) = (1-\lambda)m_t + \lambda \tanh \frac{\beta}{2} \left\{ (p_{t+1} - p_t)^2 - C \right\} \quad (4.34)$$

- The way we proceed to analyze the dynamic of above equations is:

1. Examine the homogenous population
2. Reduced dynamic when *m* is fixed



(a) $\lambda = 0.95$ (b) $\lambda = 0.25$ (c) $\lambda = 0.25$, enlarged

Dynamics for the perfect-predictor vs. static expectations binary choice system. The straight lines in the center figure represent the cross-over in preferred model. Horizontal axes are price, and vertical axes the model choice proportion m.

Analysis:

- Keep m dynamically-fixed, we get:

$$p_{t+1} = -p_t \frac{b(1-m)}{2B + b(1+m)} \quad (4.35)$$

- Then, when m>-B/b, i.e., when sufficiently agents using the more sophisticated model, convergence occurs.
- Explanation: When m is near –1, Eq.4.35 are similar to Eq.4.24, that we have introduced before, so the oscillations occurs. When m is near 1, the numerator is very small, so p_t will convergence towards 0, the equilibrium state.
- Conclusion: H_7-H_2 dynamics are very similar to H_1-H_2 case.

More complex, adaptive predictor choices

- Introduce predictive models which are the same as those in Chapter 3 in the following aspect:
  - Varying complexity
  - Same functional form
  - Contain an adaptive element
  - Agents adapt their model using observed data from the previous T price-lags.
- Difference with NR models in Chapter 3:
  - Agents consider EITHER the complexity or the number of lags, not BOTH.
  - Agents predict the behavior of the entire system.
- Process:
  - The simpler case of agents which choose a number of lags: averaging ones, equivalent to the complexity zero models of Chapter 3
  - The model-complexity choice process

Averaging predictors over varying histories

- For a given number of lags, the averaging predictor H_{av}^{T} is:

$$H_{\rm av}^{T} = \frac{1}{T}\sum_{i=0}^{T-1} p_{t-i} \qquad (4.40)$$

- We will consider pairs of these predictors, using T_1 and T_2 lags respectively:

$$H_{\rm av}^{T_1} = \frac{1}{T_1}\sum_{i=0}^{T_1-1} p_{t-i} \qquad (4.41)$$

$$H_{\rm av}^{T_2} = \frac{1}{T_2}\sum_{i=0}^{T_2-1} p_{t-i} \qquad (4.42)$$

- So the update equations for p_t, m_t are:

$$p_t = -\frac{b}{2B}\left\{(1+m_{t-1})(\frac{1}{T_1}\sum p_{t-i}) + (1-m_{t-1})(\frac{1}{T_2}\sum p_{t-i})\right\} \qquad (4.43)$$

$$m_t = (1-\lambda)m_{t-1} + \lambda \tanh\frac{\beta}{2}\left[(H_{\rm av}^{T_1} - H_{\rm av}^{T_2}) \times (2p_t - H_{\rm av}^{T_1} - H_{\rm av}^{T_2}) - C\right] \qquad (4.44)$$

Cont.

- Equations 4.43 and 4.44 can be transformed by introducing: $v = 1 - \lambda, v = -v, \varepsilon = v - 1$

$$1 = \mu \frac{b}{B}\frac{\epsilon}{2}\left[1 - \frac{\epsilon}{2}(T_1 + 2)\right] + (1-\mu)\frac{b}{B}\frac{\epsilon}{2}\left[1 - \frac{\epsilon}{2}(T_2 + 2)\right] + O(\epsilon^3) \qquad (4.47)$$

  If we solve 4.47, we will find the case which separates the stable from unstable regimes.
- Conclusion:
  - For homogenous populations(all using T_1 : for a fixed market-instability b/B, the system if unstable if $b/B>4(T_1 +2)$, and stable if $b/B<4(T_1+2)$
  - For General case: let $\alpha = (b/B)/4(T_1 +2)$. So α <1 for stability, and v>1 for instability in the T_1 only case.

    Recall the *C* is the cost of using T1(long-history predictor) relative to the cost of using T2, Let $T_2 < T_1$, consider the α <1 case, we get: $T_2<b/4B-2$, the combined system is unstable.

Adaptive predictor choice

- Now consider the more intricate case of the complexity-choice process. For a given complexity c, the adaptive predictor H^c_{adapt} is:

$$p_t^{\text{pred}} = H^c_{\text{adapt}}(p_{t-1}, \ldots, p_{t-T}) = \alpha_0 + \sum_{i=1}^{c} \alpha_i p_{t-i}, \qquad (4.57)$$

where the predictive coefficients $\alpha_i = \alpha_i(p_{t-1}, \ldots, p_{t-T})$ are functions of the past time series.

- Finally we need to solve the equations:

$$\begin{pmatrix} 1 & \langle p_{t-1}\rangle & \langle p_{t-2}\rangle & \cdots & \langle p_{t-c}\rangle \\ \langle p_{t-1}\rangle & \langle p_{t-1}^2\rangle & \langle p_{t-1}p_{t-2}\rangle & \cdots & \langle p_{t-1}p_{t-c}\rangle \\ \langle p_{t-2}\rangle & \langle p_{t-2}p_{t-1}\rangle & \langle p_{t-2}^2\rangle & \cdots & \langle p_{t-2}p_{t-c}\rangle \\ \vdots & \vdots & \vdots & & \vdots \\ \langle p_{t-c}\rangle & \langle p_{t-c}p_{t-1}\rangle & \langle p_{t-c}p_{t-2}\rangle & \cdots & \langle p_{t-c}^2\rangle \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_c \end{pmatrix} = \begin{pmatrix} \langle p_t\rangle \\ \langle p_t p_{t-1}\rangle \\ \langle p_t p_{t-2}\rangle \\ \vdots \\ \langle p_t p_{t-c}\rangle \end{pmatrix} \qquad (4.64)$$

- Lemma: the more complex models will be more sensitive as we near equilibrium: the effect on their predictions of a small change in one of the time series terms actually increases with c, for time series sufficiently near equilibrium.

Dynamics of predictors of varying complexity

- Now consider a comparison between models with different complexities:

$$H^{c_1}_{\text{adapt}} = \alpha_0 + \sum_{i=1}^{c_1} \alpha_i p_{t-i} \quad (4.65)$$

$$H^{c_2}_{\text{adapt}} = \gamma_0 + \sum_{i=1}^{c_2} \gamma_i p_{t-i} \quad (4.66)$$

- Lemma:
  - For any pair of predictors, $H_{adapt}^{c_1}$, $H_{adapt}^{c_2}$, the steady state is unstable for sufficiently large b/B, when all other parameters are fixed. And, the predictor with larger complexity is more unstable.
  - Any population of agents choosing between a pair of predictors like these will yield the usual metastable equilibrium state. More complex predictors will predict worse as we approach equilibrium.

Conclusions(I)

- The two-predictor A.R.E.D models have complex dynamics by introducing inertia.
- From the simpler models, the conflicting need of agents are:
  - Predict accurately
  - Live cheaply

  This conflict can lead to very complicated dynamics
- The authors examined predictors of varying complexity, and showed that, the more complex a predictor and agent chooses, the larger the sensitivity (and thereby instability) of the model as it neared equilibrium.

Conclusions(II)

- The remaining differences between the systems we have analyzed and those of Natural Rationality:
- NR dealt with local prediction, which little different to the global prediction we have used here
- In ARED models, there is a global, unique equilibrium, and a global price. In NR multiple equilibria exist because of the lack of global price.
- The complex models' in NR have time series which appear to be somewhat richer, while the chaotic dynamics of the simpler ARED models can contain attractors with at most 2 dimensions.

Comments

- In this chapter, the author introduce the ARED model, and try to analyze the dynamics of the phenomena in Chapter 3 by using this model. He proceed the analysis step by step:
  - The simple predictors H_1 and H_2.
  - Introduce the inertia λ to describe the nonlinear degree of the system
  - Analyze how λ influence the simple predictors
  - Introduce more sophisticated predictors H_7
  - Then the adaptive predictor by adding price lags
  - Finally add the complexity to the predictors, then observe the dynamics.
- This should give us a suggestion about how to analyze a complicate system