#### Dynamics of social networks

#### Shiwu Zhang

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### Statistical characteristics

- Diameter
  - -d: The average node's distance
- Cluster Coefficient
  - C: The average fraction of a node's neighbor pairs that are also neighbors each other
- Degree Distribution
  - -P(k): Distribution of link number for each node

# Small World vs. Scale Free

- Small World
  - Small average shortest path between two nodes scaling logarithmically with network size
  - High clustering
  - Models
    - <u>Random graphs</u>

(Taking N dots and drawing Np/2 lines between random pairs)

• Completely ordered lattice

(a low dimension regular lattice)

• <u>Watts-Strogatz model</u>

(a low dimension regular lattice with some degrees of randomness)

# Small World vs. Scale Free

- Scale Free
  - Based on connectivity distribution P(k)
    - Exponential network
    - *P(k)* follows a Possion distribution
    - Random graph, Watts-Strogatz's small world model
  - Scale-free network
    - *P(k)* follows a Power Law distribution
    - <u>Barabasi's model</u> (Growth, Preferential attachment)
    - Tolerance (*d~f*, *C~f*)
    - *f*: The fraction of nodes that is removed from network

#### Urn Transfer Model

- Urns, Balls, Pins
- At each step
  - With probability p(k+1), a new ball is added into urn1
  - With probability 1-p(k+1), an urn is selected, one ball from urn(i) is transferred to urn(i+1)
  - Including a non-preferential component
- Results
  - Power-law distribution with the exponent is greater than 2
  - References (Levene2002, Hai2003)

### Social Networks

- High cluster coefficient, short path length, scale-free connectivity
- Dynamics (nodes/links)
- Local Behaviors
- Models
  - Ebel's Transitive Model (Ebel2002, Ebel2003)

## Social Networks- Example

- Coauthorship networks
- $P(k) \sim k^{-y}$ , y=1.2 (scale free)
- Cluster coefficient
  - larger
  - Comparison with Random graph(constant linking probability/identical degree distribution)
  - Independent from degree distribution
- Path length
  - Shorter
  - Comparison with Random graph()

#### Ebel's Transitive Model

- Acquaintance network: Transitive linking (Figure 1)
- At each step
  - One individual is chosen and introduce two arbitrary acquaintances to each other
  - With probability *p*, a random person leave the networks and delete all of his links, Adding a new individual linking to existing node at random
  - Nodes number remains constant-> stationary state.
  - P << 1 (Two time scale)

#### Ebel's Transitive Model-Conclusion



Degree distribution P(k) for different values of p, which shows powerlaw distribution Cutoff at high k: finite lifetime

*P*, increasing *p*, from power-law distribution to exponential distribution

#### Ebel's Transitive Model-Conclusion

From TABLE 1

- Cluster coefficient C
  - Far higher than for a random nework
  - Insensitive to mean degree
- Path length
  - Short
  - Logarithmic scaling with system size

# Related Papers(1)

- H. Ebel et al, Dynamics of social networks. Complexity(2003)
- Davidsen *et al*, Emergence of a small world from local interactions: Modeling acquaintance networks. *PRL*, 88(12), (2002)
- M.E.J. Newman (2000). Models of the Small World.
- M. Levene et al, A stochastic model for the evolution of the web. *Computer Networks*, 39, (2002), 277-287
- H. Zhuge et al, A stochastic Growth Model for Future Web, in *Proceedings of IAT2003*, (2003)

## Related Papers (2)

- R. Albert *et al.* (2000). Error and attack tolerance of complex networks. *Nature* (406)378-382.
- A.L. Barabasi and R. Albert, (1999). Emergence of scaling in random networks. *Science* (286)509-512.
- A.L. Barabasi *et al* (2000). Scale-free characteristics of random networks: the topology of the World-wide web. Physica A (281)70-77
- R.P. Satorras and A.Vespignami, (2001). Epidemic spreading in scale-free networks. *PRL*, (86)14, p3200-3203.
- D.J. Watts and S.H. Strogatz, (1998). Collective dynamics of 'small world' networks. *Nature*, (393), p440-442.