Online Procurement Auctions for Resource Pooling in Client-Assisted Cloud Storage Systems

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***Abstract*—Latest developments in cloud computing technologies have enabled a plethora of cloud based data storage services. Cloud storage service providers are facing significant bandwidth cost as the user population scales. Such bandwidth cost can be substantially slashed by exploring a hybrid cloud storage architecture that takes advantage of under-utilized storage and network resources at storage clients. A critical component in the new hybrid cloud storage architecture is an economic mechanism that incentivizes clients to contribute their local resources, while at the same time minimizes the provider's cost for pooling those resources. This work studies online procurement auction mechanisms towards these goals. The online nature of the auction is in line with asynchronous user request arrivals in practice. After carefully characterizing truthfulness conditions under the online procurement auction paradigm, we prove that truthfulness can be guaranteed by a price-based allocation rule and payment rule. Our truthfulness characterization actually converts the mechanism design problem into an online algorithm design problem, with a marginal pricing function for resources as variables set by cloud storage service providers for online procurement auction. We derive the marginal pricing function for the online algorithm. We also prove the competitive ratio of the social cost of our algorithm against that of the offline VCG mechanism and of the resource pooling cost of our algorithm against that of the offline optimal auction. Simulation studies driven by real-world traces are conducted to show the efficacy of our online auction mechanism.**

I. Introduction

Cloud storage service (*e.g.*, Amazon S3 [1], Dropbox [2], Google Drive [3], SkyDrive [4]) has gained widespread recognition and adoption by Internet users, many of whom now routinely execute online storage and online backup tasks over the cloud. Cloud storage providers are facing two natural challenges. First, the storage service highly depends on centralized datacenters in the cloud. Accidents and natural disasters (fire, earthquake, power outage) will have significant impact on the service performance. Second, the cloud storage providers face significant cost in purchasing or renting real estate, cloud infrastructures, hardware, power and network bandwidth. To address these two challenges, the client-assisted (or user-assisted) cloud storage paradigm is a promising choice to guarantee high data availability and reliability while lowering the operational cost [5]. The client-assisted design is a complementary technology for cloud storage service that are based solely on servers in datacenters.

Recent examples of client-assisted cloud storage architecture include FS2You [6], AmazingStore [7], Wuala [8] [9], Triton [10], and Symform [11], which have examined the technical feasibility of the paradigm of client-assisted cloud storage services. However, this line of work mostly ignore the need of an incentive mechanism for users to contribute their resources, and assume that unutilized resources in storage clients can be used for client-assisted cloud storage architectures by default, or use an unattractable one by providing more cloud storage space for clients who contribute more local storage. The reality is perhaps less optimistic, as autonomous and selfish users do not have inner-incentive for contributing their resources (*e.g.*, *casual mode* in Wuala [8]). Experience from private BitTorrent communities shows that a well designed incentive mechanism can motivate users to contribute their resources and significantly improve the overall system capacity and performance [12] [13]. To help fully make client-assisted cloud storage a reality, it is imperative to design appropriate incentive mechanisms for users to engage. Online procurement auction is a natural candidate that can act as the financial catalyst for resource pooling transactions. The procurement form of the auction is due to the multi-seller (cloud users) one-buyer (storage service provider) nature of the auction, and the online property is in line with asynchronous arrivals of user bids and requests. The goals include not only optimized resource utilization across the network, but also fundamental changes in the ecosystem of cloud storage services leading to a win-win strategy for both sides. Users can receive monetary rewards for contributing their otherwise idling resources, while cloud storage providers can cut their cost substantially.

The resource pooling market is modeled as an online procurement auction market: cloud storage users are economically motivated to contribute their local resources to the storage pool. The storage pool constructed by cloud users can help to reduce the network traffic as well as storage burden at servers in datacenters. Cloud storage users are incentivized to submit selling bids to the cloud storage provider, indicating the amount of resources it plans to contribute, the time window when they are available, and the desired remuneration. Upon receiving a bid, the storage service provider makes a decision on the amount of resources to be procured, based on a pricing function. With the online procurement auction model for resource pooling market, we face the following two questions: 1) How should the mechanism guarantee the truthfulness property in such an online auction? 2) How well does our mechanism perform when compared with an offline Vickrey-Clarke-Groves (VCG) auction or an offline optimal auction?

Our general solution framework is based on converting the online procurement *auction design* problem into an online *algorithm design* problem. This conversion is based on constraints that provide conditions characterizing the online auctions' truthfulness. Hence, we first characterize the truthfulness for an online auction starting from Myerson's principle of truthfulness. We define the allocation monotonicity for online procurement auctions, and prove the existence of a critical payment scheme to guarantee truthfulness in two dimensions, *resource availability* and *resource marginal cost* under the corresponding monotone allocation rule. For convenient applications in online auction, we prove the equivalence of a price-based allocation and payment rule in guaranteeing truthfulness. With the price-based allocation rule and payment rule, we convert the online procurement auction design problem into an online algorithm design problem with marginal pricing functions as variables. An integral equation is derived to solve the marginal pricing function for guaranteeing a target competitive ratio of γ in terms of the social cost of our algorithm against that of the offline VCG mechanism, and in terms of the resource pooling cost of our algorithm against that of the offline optimal auction, and discuss the relation between the optimal target competitive ratio γ_0 and resource pool capacity, number of bids. We summarize our contributions as follows.

- We prove the equivalence of a price-based allocation rule and payment rule with those derived from Myerson's classic principle of truthfulness, for guaranteeing truthfulness in our online auction.
- With the marginal-price based allocation and payment rule, we convert an online procurement auction design into an online algorithm design problem.
- We derive an integral equation of the marginal pricing function that can guarantee our mechanism's performance with a target competitive ratio γ of the social cost of our algorithm against that of the offline VCG mechanism, and of the resource pooling cost of our algorithm against that of the offline optimal auction, and yield the range for the optimal target competitive ratio γ_0.

The rest of the paper is organized as follows. Sec. II presents related work. Sec. III describes the system model for client-assisted cloud storage systems and the online procurement auction for resource pooling. Sec. IV characterizes truthfulness for our online auctions. Sec. V presents the online algorithm design problem converted from the online auction problem, and proves the competitive ratio achieved by our algorithm. Sec. VI is performance evaluation of our algorithm under dynamic online storage clients extracted from real-world traces. Sec. VII concludes the paper.

II. Related Work

Client-assisted cloud storage paradigm design motivates much research work [6] [7] [8] [10]. Sun *et al.* [6] are the first to design, implement, and deploy a peer-assisted semi-persistent online storage system with client storage and bandwidth assistance. They make use of unstructured P2P overlay construction, sequential block scheduling mechanism, and server strategies. The design objective is to achieve a tradeoff between file availability and server cost. Yang *et al.* [7] propose a peer-assisted online storage architecture, AmazingStore, for cloud-based infrastructures. They organized nodes using DHT and gave the number of required replicas for gaining the required data availability. High fraction of data access requests are served by peers so as to reduce the server cost and improve the data availability. Mager *et al.* [8] conduct a measurement study of Wuala, a popular P2P-assisted online storage and sharing system. Toka *et al.* [14] study the impact of data placement and bandwidth allocation on the time required to complete a backup and restore operation and the clients' costs. Davoli *et al.* [10] propose solutions for acceleration on data sharing, improving the consistency, and reducing latency for agreement protocol preventing concurrent accesses on shared data. This group of work proposes P2P-assisted online storage system design or conducts measurement work on such designs based on an assumption that peers contribute their resources by default. Only Mager *et al.* [8] and Toka *et al.* [14] mention the incentive problem of peers contributing local resources. There are two modes for peers in Wuala. One type of peers are casual peers who contribute little storage and the other is storage peers who trade local storage for increased cloud storage space or reduced cloud storage cost. In this paper, we propose an online procurement auction for trading peers' local resource, which serves as an incentive mechanism for peers to contribute their local resources.

Auction mechanisms have been studied by some researchers [15] [16] and been used in some scenarios such as P2P streaming [17], WiFi pricing [18], spectrum auctions [19] [20], cloud computing pricing [21] [22] [23]. Lavi *et al.* [15] present competitive analysis for truthful online auctions. Their method is based on a threat-based approach proposed by Yaniv *et al.* [24]. Our competitive analysis is based on deriving and solving an integral equation, which is quite different from theirs. Hajiaghayi *et al.* [16] study the online auctions for reusable goods. They propose similar truthfulness conditions for online auctions, but they do not form an online algorithm design framework for the online auction mechanism design. Friedman *et al.* [18] consider extending the standard results of offline mechanism design to apply to mechanism design for online problems such as WiFi Pricing for users arriving over time. Deek *et al.* [20] study the online auction for spectrum allocation. They propose a 3D bin-packing based allocation and time-smoothed critical value based pricing scheme, and evaluate its performance through experiments. Zhang *et al.* [21] and Shi *et al.* [22] study the online auction for cloud computing resource allocation. Zhang *et al.* [21] propose the conditions of payment to ensure the truthfulness. Our method propose the allocation monotonicity, which is the necessary and sufficient condition to guarantee the truthfulness. Shi *et al.* [22] apply the threat-based approach for the competitive analysis.

Our online procurement auction design framework can be seen as an optimal online auction design, which expands Myerson's framework on optimal auction design [25].

III. Model Formulation

In a client-assisted cloud storage system, the storage service provider aims to procure its clients' unused resources to compliment server resources in its own datacenters. Such a hybrid, distributed storage architecture helps achieve higher storage cost efficiency and robustness against single point of failure. The cloud service provider needs storage and network bandwidth from clients to deploy such client-assisted storage service. Resource is assumed as a combination of storage and network bandwidth at a calculated ratio for file availability and accessibility.

Let S be the resource capacity that the service provider aims to procure during time $[0, T]$. In the online procurement auction mechanism $\mathcal{M}$, the cloud storage provider acts as the auctioneer. Storage clients on the Internet are bidders who may sell their own resources to the auctioneer. Clients are dynamic in that they may arrive and depart at any time. The set of bidders is unknown to the auctioneer *a priori*, and a bidder learns its type at the time of bidding. Consequently, the storage provider receives a stream of bids, $\mathcal{B} = (b_1, b_2, \ldots, b_j, \ldots)$, from dynamic clients, and needs to make decisions on each bid upon its submission. The bid specifies the available resource the bidder can contribute and the monetary remuneration asked in return. More specifically, the j-th received bid is a tuple $b_j = (\hat{a}_j, \hat{d}_j, \hat{Q}_j, \hat{c}_j)$, where $\hat{a}_j$ is announced arrival time of available resources, $\hat{d}_j$ is announced departure time of available resources, $\hat{Q}_j$ is announced available resource capacity, and $\hat{c}_j$ is announced marginal cost, which is the cost for bidder to provide one more unit of resource for one time unit. The type of the bidder submitting bid b_j (referred to as bidder j) is denoted by $v_j = (a_j, d_j, Q_j, c_j)$, which is private information known to bidder j only. Let $\mathcal{B}_{-j}$ denote all other bids except the j-th bid. When the auctioneer receives bid b_j, it determines the resource capacity to procure, $q_j(t, b_j, \mathcal{B}_{-j}), 0 \leq q_j \leq \hat{Q}_j$ during time $t \in [\hat{a}_j, \hat{d}_j]$, and the corresponding payment, $p_j(t, b_j, \mathcal{B}_{-j})$. We call (q_j, p_j) the allocation rule and payment rule of the online procurement auction. We will use $q_j(t, b_j)$, $p_j(t, b_j)$ instead of $q_j(t, b_j, \mathcal{B}_{-j})$, $p_j(t, b_j, \mathcal{B}_{-j})$ when there is no confusion.

Bidder j's total utility is the total payment it receives minus its total cost to offer the procured resources, integrated over the time window $[\hat{a}_j, \hat{d}_j]$: $U_j(q_j, p_j) = \int_{\hat{a}_j}^{\hat{d}_j} u_j(t, q_j, p_j)dt$, where

$$u_j(t, q_j, p_j) = p_j(t, b_j) - c_j q_j(t, b_j) \geq 0. \tag{1}$$

Eqn. (1) ensures *individual rationality* of bidders.

We are interested in mechanisms with the *truthfulness* property. We first examine the bidders' strategy space. For a_j, d_j, Q_j, bidders can not report an earlier arrival time $\hat{a}_j < a_j$, a later departure time $\hat{d}_j > d_j$, a capacity larger than what is available $\hat{Q}_j > Q_j$, as such false reports are easily detected. Hence, the bidders' strategy space is $\hat{a}_j \geq a_j$, $\hat{d}_j \leq d_j$, $\hat{Q}_j \leq Q_j$, $\hat{c}_j \neq c_j$. The definition of truthfulness for online auction mechanisms is as follows.

Definition 1 (**Truthfulness**). *An online auction mechanism $\mathcal{M}$ is truthful, if for any bidder j, regardless of the type reports of other bidders,* i.e.*, past and future bids, declaring a bid that reveals its true type can maximize its utility.* I.e.*, for a bidder with type $v_j = (a_j, d_j, Q_j, c_j)$, every bid $b_j = (\hat{a}_j, \hat{d}_j, \hat{Q}_j, \hat{c}_j)$ of the bidder satisfying $\hat{a}_j \geq a_j$, $\hat{d}_j \leq d_j$, $\hat{Q}_j \leq Q_j$, $\hat{c}_j \neq c_j$, we have*

$$U(q_j(t, v_j), p_j(t, v_j)) \geq U(q_j(t, b_j), p_j(t, b_j)). \tag{2}$$

The definition of truthfulness for our model is two-fold: one is to reveal resource availability, *i.e.*, a_j, d_j, Q_j, truthfully; the other is to reveal resource cost, *i.e.*, c_j, truthfully.

Let $\mathcal{B}_t$ be the set of stream bids received by time t, $\mathcal{B}_t = \{b_j | \hat{a}_j \leq t\}$. With $\mathcal{B}_t$, we can define the service provider's completeness ratio for any time t''s resource pooling after receiving streaming bids $\mathcal{B}_t$, $R(t', t)$,

$$R(t', t) = \frac{\sum_{j \in \mathcal{B}_t} q_j(t', b_j)}{S} \in [0, 1], t \leq t' \leq T. \tag{3}$$

When the required capacity S is not achieved by time t through client-assisted resources at time t, *i.e.*, $R(t, t) < 1$, it will need to use the storage and bandwidth from servers in the datacenters to compensate. Let c_s be the marginal resource cost from servers.

The cost of the cloud provider for providing a capacity of S resource under a bidding sequence $\mathcal{B}$ under mechanism $\mathcal{M}$: (q_j, p_j) is denoted by $C_{\mathcal{M}}(\mathcal{B})$. The resource pooling cost is the total payment he pays for resource pooling plus the server cost for compensating the remaining resource.

$$C_{\mathcal{M}}(\mathcal{B}) = \sum_{j \in \mathcal{B}} \int_{\hat{a}_j}^{\hat{d}_j} p_j(t, b_j)dt + \int_0^T c_s \cdot S[1 - R(t, t)]dt. \tag{4}$$

The online procurement auction mechanism design problem can be modeled as an online optimization problem as follows,

$$\begin{aligned} \min \quad & C_{\mathcal{M}}(\mathcal{B}) \\ \text{s.t.} \quad & (1)(2)(3) \end{aligned}$$

The objective function $C_{\mathcal{M}}(\mathcal{B})$ can be rearranged as follows:

$$C_{\mathcal{M}}(\mathcal{B}) = \int_0^T [\sum_{j \in \mathcal{B}_t} p_j(t, b_j) + c_s \cdot S[1 - R(t, t)]]dt$$

And the constraints about the allocation rule and payment rule are also decoupled at different time points. The optimization problem is equivalent to minimizing the following problem for any time $t \in [0, T]$,

$$\begin{aligned} \min \quad & \sum_{j \in \mathcal{B}_t} p_j(t, b_j) + c_s \cdot S[1 - R(t, t)] \\ \text{s.t.} \quad & (1)(2) \\ & R(t, t) \in [0, 1]. \end{aligned} \tag{5}$$

Optimization problem (5) can be seen as a sub-problem of resource pooling for time $t \in [0, T]$. Table I summarizes important notations for ease of reference.

TABLE I: Important Notations

S	resource pool capacity.	T	time span of resource.
$W_{\mathcal{M}}(\mathcal{B})$	the total social cost.	$\mathcal{B}$	stream of bids.
$\mathcal{M}$	the online procurement auction mechanism.		
$\mathcal{B}_{-j}$	all other bids except the j-th bid.		
b_j	the j-th bid in the bid stream.		
v_j	true type of the j-th bid.		
a_j	true arrival time of bidder j's resource.		
d_j	true departure time of bidder j's resource.		
Q_j	true available resource of bidder j.		
c_j	true unit resource cost for unit time of bidder j.		
$q_j(t, b_j)$	resource capacity procured from bidder j.		
$p_j(t, b_j)$	payment to bidder j for time t's resource.		
$U_j(q_j, p_j)$	bidder j's utility.		
$\mathcal{B}_t$	streaming bids received by time t.		
$R(t', t)$	completeness ratio of time t''s resource pooling based on procurement by t.		
c_s	unit resource cost for unit time of storage provider.		
$C_{\mathcal{M}}(\mathcal{B})$	cloud storage provider's total cost.		

IV. Truthfulness Characterization for Online Auctions

A. Revisiting Myerson's Principle of Truthfulness

Myerson [25] formulated the classic characterization of offline truthful mechanisms in a general Bayesian setting, for auctions of a single indivisible good.

Lemma 1 (Myerson, 1981). *Let $P_i(b_i)$ be the probability of bidder i with bid b_i winning an auction. A mechanism is truthful if and only if the followings hold for a fixed $\mathbf{b}_{-i}$:*

- *$P_i(b_i)$ is monotonically non-decreasing in b_i;*
- *Bidder i bidding b_i is charged $b_i P_i(b_i) - \int_0^{b_i} P_i(b)db$*

Gopinathan *et al.* [26] pointed out the two interpretations of Myerson's principle: (i) there exists a minimum bid b_i' such that i will win only if it bids at least b_i', and (ii) the payment charged to i for a fixed $\mathbf{b}_{-i}$ is independent of b_i.

We will first present the truthfulness characterization for our online procurement auction by adapting and extending Myerson's principle of truthfulness. In our online procurement auction, to guarantee truthfulness is to guarantee that bidders bid their true types in both resource availability and resource marginal cost, *i.e.*, $b_j = v_j(\hat{a}_j = a_j, \hat{d}_j = d_j, \hat{Q}_j = Q_j, \hat{c}_j = c_j)$, regardless of past and future streams of other bids.

Myerson's principle of truthfulness first points out the monotonicity criterion of allocation rule $P_i(b_i)$ under a Bayesian game model for auctions of a single indivisible good. Our online procurement auction is an online reverse auction for a divisible good with quantity S. Hence, we first define a corresponding allocation monotonicity for our auction, which is necessary and sufficient for the existence of a payment rule eliciting truthful bids.

Definition 2 (Allocation Monotonicity). *Consider two bids $b_j' = (a_j', d_j', Q_j', c_j')$ and $b_j = (\hat{a}_j, \hat{d}_j, \hat{Q}_j, \hat{c}_j)$ in the online procurement auction. If $\hat{a}_j \le a_j', \hat{d}_j \ge d_j'$, $\hat{Q}_j \ge Q_j'$, $\hat{c}_j \le c_j'$,* i.e., *bid b_j''s resource availability is a subset of that of bid b_j, and bid b_j''s resource marginal cost is no smaller than that of bid b_j, we say bid b_j dominates bid b_j', denoted by $b_j \succeq b_j'$. An allocation rule q is monotone if*

$$\int_{\hat{a}_j}^{\hat{d}_j} q_j(t, b_j)dt \ge \int_{a_j'}^{d_j'} q_j(t, b_j')dt \quad \forall j, \text{ if } b_j \succeq b_j'. \tag{6}$$

Let us examine the difference between the allocation monotonicity criterion between Myerson's principle and our definition. Myerson considers a bayesian game model for auctions of one single indivisible good. Hence, the allocation probability to one bidder when having its bid b_i, $P_i(b_i)$, is used to represent the allocation rule. This allocation probability is used because we consider other bids satisfy a probability distribution and the probability distribution of other bids is a priori. In our online procurement auction for divisible good with quantity S, we use the allocation quantity to one bidder when having its bid b_j, $\int_{\hat{a}_j}^{\hat{d}_j} q_j(t, b_j)dt$, to represent the allocation rule, given other bids are fixed. When other bids vary according to a priori probability distribution, the expectation of the allocation quantities is still monotone with b_j. This is consistent with $P_i(b_i)$'s monotonicity in a single indivisible good's auction.

Given allocation monotonicity, we present the payment scheme to implement a truthful online procurement auction in Theorem 1.

Theorem 1. *There is a payment rule p such that the mechanism $\mathcal{M} : (q, p)$ is truthful if and only if the allocation rule q is monotone. And the payment scheme can be expressed as follows,*

$$p_j(t, b_j) = \hat{c}_j \cdot q_j(t, b_j) + \int_{\hat{c}_j}^{+\infty} q_j(t, (\hat{a}_j, \hat{d}_j, \hat{Q}_j, x))dx. \tag{7}$$

Theorem 1 can be seen as an extension of Myerson's principle of truthfulness to online procurement auctions of a divisible good. We prove the truthfulness of resource availability and resource marginal cost under conditions in Theorem 1. Details of its proof are in Appendix IX-A.

B. A Price-based Allocation and Payment Rule

Theorem 1's characterization of truthfulness stands in the viewpoint of the allocation rule. Once the allocation rule is determined, the payments are correspondingly determined. This characterization is convenient for offline auctions where all bids are collected simultaneously and then the allocation decisions for all related bids are made at the same time. In such cases, the allocation rule is obvious. However, this does not apply to online auctions where bids arrive at different times, with allocation decisions made at realtime.

We will present an equivalent condition to guarantee truthfulness of online procurement auction mechanisms, in which a marginal pricing function will be introduced. The allocation and payment are based on the pricing function.

Price-based Allocation and Payment: Let $\Psi(t, R)$ be the marginal pricing function of the cloud storage provider for procuring time t's resources from clients when the completeness ratio of time t's resource pooling is R. It is easy to see that $\Psi(t, R)$ is bid-independent .

In an online procurement auction for S resource capacity, when the cloud storage provider receives a bid $b_j = (\hat{a}_j, \hat{d}_j, \hat{Q}_j, \hat{c}_j)$, it procures $q_j(t, b_j)$ resource from the bidder and pays $p_j(t, b_j)$ to the bidder. $q_j(t, b_j)$ and $p_j(t, b_j)$ are determined as follows,

$$q_j(t, b_j) = \begin{cases} 0, & \text{if } \hat{c}_j \geq \Psi(t, R); \\ \min\{\hat{Q}_j, S \cdot [\Psi^{-1}(t, \hat{c}_j) - R(t, \hat{a}_j^-)]\}, \\ & \text{if } \hat{c}_j < \Psi(t, R). \end{cases} \quad (8)$$

$$p_j(t, b_j) = \int_{R(t,\hat{a}_j^-)}^{R(t,\hat{a}_j)} S \cdot \Psi(t, R) dR \quad (9)$$

Here, $R(t, \hat{a}_j^-) = \sum_{k \in \mathcal{B}_{\hat{a}_j^-}} q_k(t, b_k)/S$ is the completeness ratio for time t's resource pooling, just before receiving bid b_j.

We have Theorem 2 for the truthfulness of price-based online procurement auctions.

Theorem 2. *When a cloud storage provider determines its allocation and payment according to Eqn. (8) and (9) when receiving a bid, where $\Psi(t, R)$ is non-increasing in completeness ratio R, the mechanism $\mathcal{M} : (q, p)$ is truthful.*

Proof: We first prove monotonicity of the allocation rule. Given two bids $b_i = (\hat{a}_i, \hat{d}_i, \hat{Q}_i, \hat{c}_i)$, $b_i' = (a_i', d_i', Q_i', c_i')$, $b_i \succeq b_i'$ from a bidder i, we will prove $q_i(t, b_i) \geq q_i(t, b_i')$.

Let us examine the allocation under a new bid $\widetilde{b}_i = (a_i', d_i', \hat{Q}_i, \hat{c}_i)$. It is easy to verify that $b_i \succeq \widetilde{b}_i \succeq b_i'$.

As the resource availability of b_i dominates $\widetilde{b}_i$, we have $\hat{a}_i \leq a_i'$. As bid $\widetilde{b}_i$ arrives later, the completeness ratio of time t's resource pooling before receiving bid b_i is smaller than or equal to that before receiving bid $\widetilde{b}_i$, *i.e.*,

$$S \cdot R(t, \hat{a}_i^-) = \sum_{k \in \mathcal{B}_{\hat{a}_i^-}} q_k(t, b_k) \leq \sum_{k \in \mathcal{B}_{a_i'^-}} q_k(t, b_k) = S \cdot R(t, a_i'^-)$$

The available resource and resource marginal cost is the same for b_i and $\tilde{b}_i$. According to Eqn. 8, we have $q_i(t, b_i) \geq q_i(t, \widetilde{b}_i)$.

As for bid b_i' and $\widetilde{b}_i$, the resource arrival times are the same for the two bids. Hence, the completeness ratio of time t's resource pooling when receiving bid b_i' or $\widetilde{b}_i$ are the same. As $\Psi(t, R)$ is non-increasing with R, we can verify that its inverse function $\Psi^{-1}(t, c)$ is also non-increasing with c. Because $\hat{c}_i \leq c_i'$, we have

$$\Psi^{-1}(t, \hat{c}_i) \geq \Psi^{-1}(t, c_i')$$

We also have $\hat{Q}_i \geq Q_i'$, this results in $q_i(t, \widetilde{b}_i) \geq q_i(t, b_i')$ according to Eqn. (8).

In conclusion, we proved $q_i(t, b_i) \geq q_i(t, b_i')$. The allocation rule based on the marginal pricing function $\Psi(t, R)$is monotone. With the monotone allocation rule, based on Theorem 1, we know that the payment rule in Eqn. (7) truthfully implements the allocation rule. The price-based payment Eqn. (9) can be derived by rearranging Eqn. (7) based on the price-based allocation Eqn. (8). ■

V. A Competitive Online Procurement Algorithm

The price-based allocation rule and payment rule work in concert to not only guarantee the truthfulness of an online auction mechanism, but also help us convert an online auction design problem into an online algorithm design problem. The online algorithm design problem has the marginal pricing function as its variable. Using an offline VCG auction and an offline optimal auction as benchmarks for social cost and cloud storage provider's pooling cost respectively, we derive an integral equation for guaranteeing a target competitive ratio γ, and further derive the marginal pricing function from the integral equation.

A. Online Algorithm Design Problem Converted from Online Auction Design

By substituting the price-based payment rule (9) into the optimization problem (5), we obtain the following online algorithm design problem,

$$\begin{aligned} \min \quad & \sum_{j \in \mathcal{B}_t} \int_{R(t,\hat{a}_j^-)}^{R(t,\hat{a}_j)} \Psi(t, R) S \cdot dR + c_s \cdot S[1 - R(t, t)] \\ \text{s.t.} \quad & \Psi(t, R) \text{ is a non-increasing function} \\ & \Psi(t, 1) \leq c_{min} \end{aligned} \quad (10)$$

Condition (2) of optimization problem (5), *i.e.*, property of truthfulness, can be guaranteed based on the price-based allocation and payment rule. $\Psi(t, 1) \leq c_{min}$ guarantees condition $R(t, t) \in [0, 1]$ of (5). Condition (1) is satisfied as the price-based allocation rule and payment rule only procure resource from clients with resource marginal cost for time t smaller than $\Psi(t, R)$, and pay clients prices that are higher than the resource marginal cost. Hence, for any bidder j, $U_j(q_j, p_j) \geq 0$.

We summarize our algorithm framework for the online procurement auction in Algorithm 1.

B. Marginal Pricing Function Derivation and Competitive Ratio Analysis

To implement Algorithm 1, we need to give the marginal pricing function. We will see the pricing function will affect the competitive ratio of our online procurement auction. In our online procurement auction design, we aim at minimizing the cloud provider's resource pooling cost, which is the summation of the payment of the cloud storage provider for procuring resource from clients and the cost of providing complimentary resource from servers. With the social cost of offline VCG auction as a lower bound of the resource pooling cost of the offline optimal auction, we analyze the competitive ratio of the resource pooling cost against that of the offline optimal auction. Besides the resource pooling cost, another important metric is the social cost, *i.e.*, the total cost for clients and servers to provide S resource. We benchmark our online algorithm against the offline VCG auction that achieves the minimum social cost.

Algorithm 1 Cloud Storage Provider's Algorithm for the Online Procurement Auction

Input: Marginal pricing function $\Psi(t,R)$, S, T.
Output: $q_j(t,b_j)$, $p_j(t,b_j)$

```
Initialize the completeness ratio of time t ∈ [0,T]'s resource pooling R_t = 0
while Receive a bid b_j do
    for t ∈ [â_j, d̂_j] do
        if R_t = 1 then
            Continue
        end if
        if ĉ_j < Ψ(t, R_t) then
            if Ψ^{-1}(t, ĉ_j) > 1 then
                Set Ψ^{-1}(t, ĉ_j) = 1
            end if
            Procure q_j(t, b_j) resource from bidder j according to Eqn.(8)
            Pay p_j(t, b_j) to bidder j according to Eqn.(9)
            Update R_t = R_t + q_j(t, b_j)/S
        end if
    end for
end while
```

The social cost for providing time t's S resource under our online algorithm is

$$W_{\mathcal{M}}(t,\mathcal{B}) = \sum_{j\in\mathcal{B}_t} c_j q_j(t,b_j) + c_s \cdot S[1-R(t,t)]. \tag{11}$$

The total social cost is the integration of $W_{\mathcal{M}}(t,\mathcal{B})$ among time $[0,T]$, *i.e.*, $W_{\mathcal{M}}(\mathcal{B}) = \int_0^T W_{\mathcal{M}}(t,\mathcal{B})dt$.

Theorem 3. *When the cloud storage provider uses the marginal pricing function as $\Psi(t,R) = c_s - c_s(1-\frac{1}{\gamma})e^{\frac{R}{\gamma}}$, where R is the completeness ratio of procured resource for time t, our price-based online procurement auction will achieve the resource pooling cost no larger than γ times that of an offline optimal auction,* i.e., *$C_{\mathcal{M}}(\mathcal{B}) \le \gamma \cdot C_{opt}(\mathcal{B})$ and achieve the social cost no larger than γ times that of the offline VCG auction,* i.e., *$W_{\mathcal{M}}(\mathcal{B}) \le \gamma \cdot W_{vcg}(\mathcal{B})$. The optimal γ should be among the range of $[\gamma_{0,min}, \gamma_{0,max}]$. Here $\gamma_{0,min}$ and $\gamma_{0,max}$ are the solutions to equations $(1-\frac{1}{\gamma}) \cdot e^{\frac{1}{\gamma}} = 1 - \frac{c_{max}}{c_s}$, $(1-\frac{1}{\gamma}) \cdot e^{\frac{1}{\gamma}} = 1 - \frac{c_{min}}{c_s}$ respectively. $C_{opt}(\mathcal{B})$, $W_{vcg}(\mathcal{B})$ is the cloud storage provider's resource pooling cost under offline optimal auction and the social cost under offline VCG auction respectively.*

Proof: We first check the social cost under our online procurement auction and offline VCG auction. Given some bidding sequence $\mathcal{B}$, let $q_j(t,b_j)$ be the quantity procured from bidder j for pooling time t's resource, and $\Psi(t,R(t,\hat{a}_j))$ the lowest procurement price for pooling time t's unit resource of unit time from bidder j. Let l be the last bidder with resource procured, the corresponding lowest procurement price for pooling time t's resource from bidder l is $\Psi(t,R(t,\hat{a}_l))$. Let $R(t,\hat{a}_l)$ be the final completeness ratio for time t's resource procurement after procuring bidder l's resource.

The pooling cost for time t's resource is

$$C_{\mathcal{M}}(t,\mathcal{B}) = \int_0^{R(t,\hat{a}_l)} \Psi(t,R)S \cdot dR + c_s \cdot [S - S \cdot R(t,\hat{a}_l)]$$

When the cloud provider procures resource, it pays more than the bidders' cost. Hence, the social cost for providing S resource will be no larger than the cloud provider's resource pooling cost, *i.e.*, $W_{\mathcal{M}}(t,\mathcal{B}) \le C_{\mathcal{M}}(t,\mathcal{B})$.

An offline auction mechanism can collect all bids $\mathcal{B} = (b_1, b_2, \ldots, b_j, \ldots)$, then make allocation and payment decisions. As in our procurement auction, servers' resource can be utilized as a reserved backup resource. Correspondingly, we add a new bid b_0 with announced cost c_s and unbounded resource capacity.

For procuring time t's resource, let ($b_1^{(vcg)}$, $b_2^{(vcg)}$, $b_3^{(vcg)}$, $\ldots$, $b_x^{(vcg)}$) be the ascending sequence of procured bids according to the announced cost under offline VCG auction. The price paid to bidders is no smaller than the announced cost of bid $b_x^{(vcg)}$. As the offline VCG auction procure the total S capacity, this means the total resource capacity of the bids procured in offline VCG auction is no smaller than the procured resource capacity from clients in our online procurement auction. Hence, the announced cost of b_x^{vcg} is larger than or equal to $\Psi(t,R(t,\hat{a}_l))$. Otherwise, all bids in $(b_1^{(vcg)}, b_2^{(vcg)}, b_3^{(vcg)}, \ldots, b_x^{(vcg)})$ have announced cost smaller than $\Psi(t,R(t,\hat{a}_l))$. our online algorithm will procure more than S resource capacity. Hence, the social cost under offline VCG auction for providing time t's resource is $W_{vcg}(t,\mathcal{B}) \ge S \cdot c_x^{(vcg)} \ge S \cdot \Psi(t,R(t,\hat{a}_l))$. Here $c_x^{(vcg)}$ is the announced cost of bid $b_x^{(vcg)}$.

With the social cost for providing time t's resource under our online procurement auction and offline VCG auction, the competitive ratio in terms of the total social cost can be calculated as follows,

$$\begin{aligned}\frac{W_{\mathcal{M}}(\mathcal{B})}{W_{vcg}(\mathcal{B})} &= \frac{\int_0^T W_{\mathcal{M}}(t,\mathcal{B})}{\int_0^T W_{vcg}(t,\mathcal{B})} \\ &\le \frac{\int_0^T [\int_0^{R(t,\hat{a}_l)} \Psi(t,R)S \cdot dR + c_s \cdot (S - S \cdot R(t,\hat{a}_l))]dt}{\int_0^T [S \cdot \Psi(t,R(t,\hat{a}_l))]dt}\end{aligned}$$

To guarantee a competitive ratio of γ, we can let

$$\frac{\int_0^{R(t,\hat{a}_l)} \Psi(t,R)S \cdot dR + c_s \cdot (S - S \cdot R(t,\hat{a}_l))}{S \cdot \Psi(t,R(t,\hat{a}_l))} = \gamma \tag{12}$$

By solving the integral equation (12), we can derive the expression of $\Psi(t,R)$ as:

$$\Psi(t,R) = c_s - c_s(1-\frac{1}{\gamma}) \cdot e^{\frac{R}{\gamma}}$$

Let us consider the bound condition of the marginal pricing function:

(i) If $\Psi(t,1) < c_{min}$, this means $\Psi(t,R)$ will becomes equal to c_{min} at a point $R < 1$, and the cloud storage provider will stop procuring resources from bidders as the completeness ratio does not reach 1. This may waste the chances to procure resource with lower announced cost. use lower cost than server cost to buy coming bidders' resources. Hence, this is not optimal for setting target competitive ratio, γ.

(ii) If $c_{max} > \Psi(t,1) > c_{min}$, this means the marginal pricing function will not be continuous at $R = 1$. The cloud storage provider needs to set the price as c_{min} when $R = 1$. But the useful scenarios for setting $c_{max} > \Psi(t, 1^-) > c_{min}$ is when the number of bidders is small enough or target resource pool capacity S is large enough. In such scenarios, R can not reach 1 due to scare resource from storage clients. Hence, the optimal choice is to accept as many bids as possible. The optimal bound condition is setting $\Psi(t,1) = c_{max}$. $(1-\frac{1}{\gamma})\cdot e^{\frac{1}{\gamma}} = 1 - \frac{c_{max}}{c_s}$. We can get the optimal target competitive ratio is $\gamma_{0,min} = \frac{1}{W_0(\frac{c_{max}/c_s - 1}{e})+1}$, here, $W_0(x)$ is a Lambert W function and $W_0(x) \geq -1$.

(iii) If the number of bids is large enough or target resource pool capacity S is small enough, R can easily reach 1, the optimal solution is setting $\Psi(t,1) = c_{min}$. The optimal target competitive ratio $\gamma_{0,max} = \frac{1}{W_0(\frac{c_{min}/c_s - 1}{e})+1}$.

Summarizing the above analysis, we can see the optimal target competitive ratio is in the range $\gamma_0 \in [\gamma_{0,min}, \gamma_{0,max}]$.

Next, let us see the resource pooling cost under our online procurement auction and offline optimal auction.

As the cloud storage provider pays more than bidders' announced cost for procuring resource from clients, the resource pooling cost under the offline optimal auction should be larger than or equal to the social cost under the offline VCG auction, *i.e.*, we have $C_{opt}(t,\mathcal{B}) \geq W_{vcg}(t,\mathcal{B}) \geq S \cdot \Psi(t, R(t,\hat{a}_l))$.

Hence, the competitive ratio of our online procurement auction, in terms of resource pooling cost, is:

$$\frac{C_{\mathcal{M}}(\mathcal{B})}{C_{opt}(\mathcal{B})} = \frac{\int_0^T C_{\mathcal{M}}(t,\mathcal{B})}{\int_0^T C_{opt}(t,\mathcal{B})}$$
$$\leq \frac{\int_0^T [\int_0^{R(t,\hat{a}_l)} \Psi(t,R)S \cdot dR + c_s \cdot (S - S \cdot R(t,\hat{a}_l))]dt}{\int_0^T [S \cdot \Psi(t, R(t,\hat{a}_l))]dt}$$

which equals the competitive ratio γ of social cost. ■

VI. Performance Evaluations

In this section, we study the social cost and resource pooling cost achieved by our online procurement algorithm in the context of real-world traces.

A. Experiment Settings

The clients' arrival/departure time and available bandwidth are extracted from real-world traces [27]. We use the session 29 trace for client level data collected from `transamrit.net` with 5 minute sampling intervals, and plot the online statistical information in Fig. 1.

By default, the cost of clients' unit resource for a unit time is generated randomly in the range of $[0.01, 0.05]$\$/GB per hour. The cloud storage provider's server cost for providing resource is 0.1\$/GB per hour. The total resource pooling capacity of the cloud storage provider is $50KB/s$. We consider a time range of 7 days. The number of bids is 7000.

We compare the social cost achieved by our online procurement with that of the offline VCG auction. We compare the resource pooling cost by our online procurement with that of optimal auction indirectly. As the resource pooling cost of offline optimal auction should be larger than the social cost achieved by offline VCG auction, we use the social cost achieved by offline VCG auction as a lower bound for it. Hence, in our experiments, the social cost and resource pooling cost of our online procurement auction are both compared with the social cost of the offline VCG auction.

B. Performance of Our Online Procurement Mechanism

Fig. 2 compares the resource pooling cost and social cost of our online procurement auction mechanism with offline auctions. We make comparisons under different resource pooling capacity, number of bids, and ratio of server cost and average bidding cost. We use the target competitive ratio $\gamma_{0,max}$ in the marginal pricing function. Fig. 2a varies the resource capacity among $[10, 100]KB/s$. Fig. 2b varies the number of bids among $[1000, 7000]$. Fig. 2c varies the ratio between marginal server cost c_s and average bidding cost among $[2, 6]$. Our online procurement auction achieves the social cost and resource pooling cost within the target competitive ratio. In most cases, our online procurement auction's resource pooling cost is lower than that of the offline VCG auction.

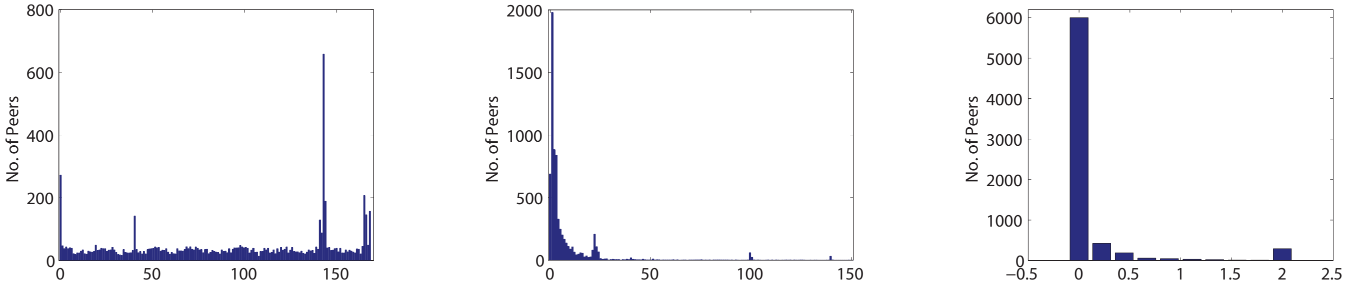
C. How Does Performance Change with γ?

Fig. 3 & Fig. 4 show the change of real cost ratio with the target competitive ratio. The black star lines in Fig. 3a & Fig. 3b are the position of $\gamma_{0,min}$. In Fig. 3a, $S = 5KB/s$, which means clients' resource is enough, we can see as the target competitive ratio equals $\gamma_{0,max}$, our online procurement auction achieves the optimal competitive ratio. In Fig. 3b, $S = 80KB/s$, which means clients' resource is not enough, we can see the real cost ratio is not sensitive to the target competitive ratio. These two figures verify the optimal target competitive ratio is among $[\gamma_{0,min}, \gamma_{0,max}]$.

Fig. 3c, Fig. 3d and Fig. 4 show the real ratio for resource pooling (upper one) and the real ratio for social cost (lower one) under different target competitive ratio and changing resource pool capacity, number of bids, and ratio of server cost and average bidding cost, ratio of the maximum and minimum bidding cost. We can see when our algorithm achieves the best performance, the target competitive ratio is no larger than $\gamma_{0,max}$. As the target competitive ratio becomes small enough, the real cost ratio may even become higher, which means the target competitive ratio should not be too small. As S is large enough, or number of bids is small enough, the real performance is insensitive to γ.

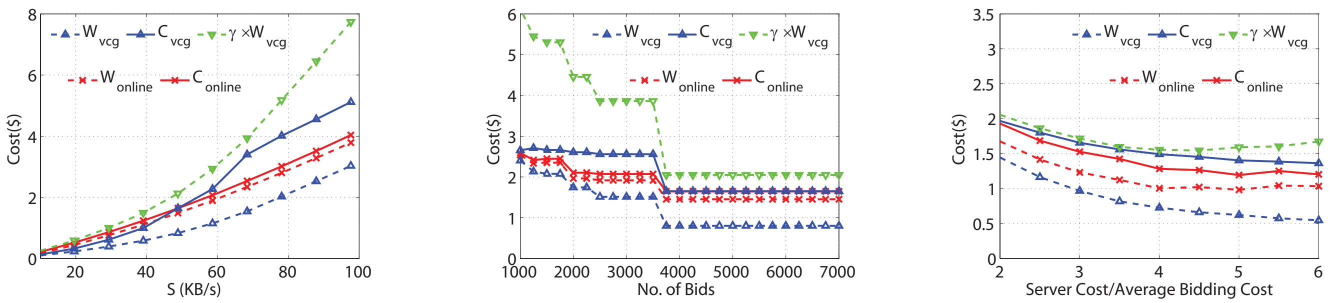
VII. Conclusions

Client-assisted cloud storage is emerging as a new, exciting hybrid architecture for online storage systems that represent a potential win-win solution for both cloud storage providers and cloud users. While recent research along this thread have focused on technical challenges, this work represents the first effort that examines the economic side of the picture. We design an online procurement auction for resource pooling by the provider, which serves as a financial catalyst for bringing



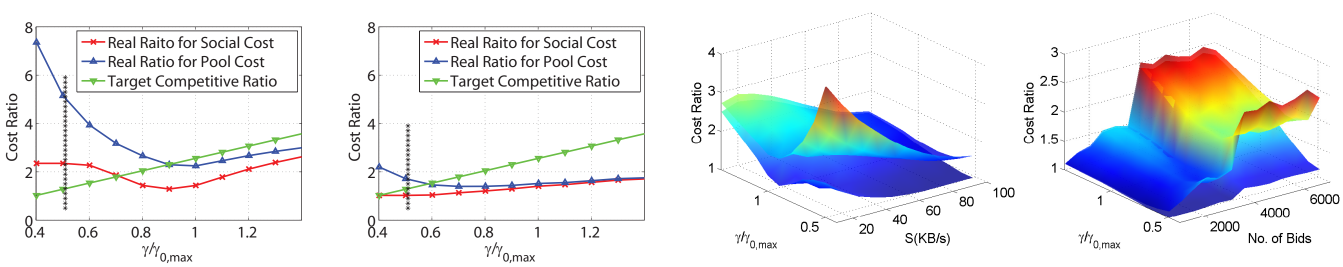
(a) Arrival Time Distribution (days) (b) Online Time Distribution (hours) (c) Resource Capacity Distribution (MB/s)

Fig. 1: Statistical Information of Client Online Behavior



(a) (b) (c)

Fig. 2: Performance of our online procurement auction under varying resource pooling capacity S, No. of bids, and ratio among server cost and average bidding cost



(a) Real Cost Ratio, S=5KB/s (b) Real Cost Ratio, S=80KB/s (c) Real Cost Ratio under Varying Pooling Capacity (d) Real Cost Ratio under Varying No. of Bids

Fig. 3: Real cost ratio with varying γ

client-assisted cloud storage into reality. The auction proposed is truthful and guarantees a provable competitive ratio against optimal offline algorithms.

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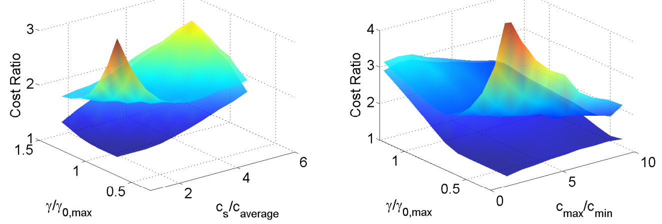

(a) Real Cost Ratio vs. Varying Ratio among Server Cost and Average Bidding Cost

(b) Real Cost Ratio vs. Varying Ratio among Maximum Bidding Cost and Minimum Bidding Cost

Fig. 4: Real cost ratio under varying γ and ratio among server cost and bidding cost

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IX. Appendix

A. Proof of Theorem 1

Proof: (a) We first prove the "if" part. Let q be a monotone allocation rule and consider a bid $b_j = (a_j, d_j, Q_j, c_j)$. We show that the allocation rule q in combination with the payment rule p in Eqn. 7 constitute a truthful mechanism. We prove the truthfulness for resource availability and resource cost respectively.

(i) We first prove the truthfulness for resource cost revelation, *i.e.*, bidders bidding their true cost will maximize their utility. We prove it by contradiction. If the mechanism is not truthful, there is a bidder i, with a true type $v_i = \{a_i, d_i, Q_i, c_i\}$, and a non-truthful bid $b_i = (\hat{a}_i, \hat{d}_i, \hat{Q}_i, \hat{c}_i)$, $b_i \neq v_i$ such that the utility $U[q_i(t, b_i), p_i(t, b_i)]$ of bidder i if it bids b_i is strictly greater than the utility $U[q_i(t, v_i), p_i(t, v_i)]$, which can be written as follows,

$$\int_{\hat{a}_i}^{\hat{d}_i} \left[(\hat{c}_i - c_i) q_i(t, b_i) + \int_{\hat{c}_i}^{+\infty} q_i(t, (\hat{a}_i, \hat{d}_i, \hat{Q}_i, x)) dx \right] dt$$
$$> \int_{a_i}^{d_i} \int_{c_i}^{+\infty} q_i(t, (a_i, d_i, Q_i, x)) dx dt$$

As the allocation rule is monotone, and $\hat{a}_i \geq a_i$, $\hat{d}_i \leq d_i$, $\hat{Q}_i \leq Q_i$, we have

$$\int_{a_i}^{d_i} \int_{c_i}^{+\infty} q_i(t, (a_i, d_i, Q_i, x)) dx dt$$
$$\geq \int_{\hat{a}_i}^{\hat{d}_i} \int_{c_i}^{+\infty} q_i(t, (\hat{a}_i, \hat{d}_i, \hat{Q}_i, x)) dx dt$$

With the above two inequalities, we get the following relationship,

$$\int_{\hat{a}_i}^{\hat{d}_i} (\hat{c}_i - c_i) q_i(t, b_i) dt > \int_{\hat{a}_i}^{\hat{d}_i} \int_{c_i}^{\hat{c}_i} q_i(t, (\hat{a}_i, \hat{d}_i, \hat{Q}_i, x)) dx dt$$

Hence, if $\hat{c}_i > c_i$, by dividing both sides by $\hat{c}_i - c_i$ we get $\int_{\hat{a}_i}^{\hat{d}_i} q_i(t, b_i) dt$ is strictly larger than the average of $\int_{\hat{a}_i}^{\hat{d}_i} q_i(t, (\hat{a}_i, \hat{d}_i, \hat{Q}_i, x)) dt$ over $x \in [c_i, \hat{c}_i]$, which contradicts the monotonicity of q. If $\hat{c}_i < c_i$, by dividing both sides by $\hat{c}_i - c_i$, we get $\int_{\hat{a}_i}^{\hat{d}_i} q_i(t, b_i) dt$ is strictly less than the average of $\int_{\hat{a}_i}^{\hat{d}_i} q_i(t, (\hat{a}_i, \hat{d}_i, \hat{Q}_i, x)) dt$ over $x \in [\hat{c}_i, c_i]$, which again contradicts the monotonicity of q. This contradiction proves the truthfulness of the mechanism $\mathcal{M} : (q, p)$ for resource cost.

(ii) Secondly, we prove the truthfulness for resource availability. As we have prove the truthfulness for resource cost, *i.e.*, for any bid $b_i = (\hat{a}_i, \hat{d}_i, \hat{Q}_i, \hat{c}_i)$, the bid $b_i' = (\hat{a}_i, \hat{d}_i, \hat{Q}_i, c_i)$ with the same resource availability and the true cost value will achieve a larger utility. Next we will prove the utility under bid v_i will be larger than that under bid b_i'.

It is obvious that

$$U[q_i(t, v_i), p_i(t, v_i)] = \int_{a_i}^{d_i} \int_{c_i}^{+\infty} q_i(t, (a_i, d_i, Q_i, x)) dx dt$$
$$\geq \int_{\hat{a}_i}^{\hat{d}_i} \int_{c_i}^{+\infty} q_i(t, (\hat{a}_i, \hat{d}_i, \hat{Q}_i, x)) dx dt = U[q_i(t, b_i'), p_i(t, b_i')]$$

Hence, we prove the truthfulness for resource availability.

(b) We then prove the "only if" part. We have a truthful mechanism $\mathcal{M} : (q, p)$. Consider a bidder i and its two possible types $v_i = (a_i, d_i, Q_i, c_i)$, $v_i' = (a_i', d_i', Q_i', c_i')$. $v_i \succ v_i'$, and $\mathcal{B}_{-i}$ are the same. If the scenario is that v_i is bidder i's true type, according to the truthfulness, we have

$$\int_{a_i}^{d_i} [p_i(t, v_i) - c_i q_i(t, v_i)] dt \geq \int_{a_i'}^{d_i'} [p_i(t, v_i') - c_i q_i(t, v_i')] dt$$

If the scenario is that v_i' is bidder i's true type, we have

$$\int_{a_i'}^{d_i'} [p_i(t, v_i') - c_i' q_i(t, v_i')] dt \geq \int_{a_i}^{d_i} [p_i(t, v_i) - c_i' q_i(t, v_i)] dt$$

By adding the two inequalities above and using $c_i' > c_i$, we have $\int_{a_i}^{d_i} q_i(t, v_i) dt > \int_{a_i'}^{d_i'} q_i(t, v_i') dt$. Therefore q is monotone. ∎