Sparse-Partial Wavelength Conversion: Converter Placement and Wavelength Assignment[*]

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ABSTRACT

Wavelength conversion has been shown as one of the key techniques to improve blocking performance in a wavelength-routed all-optical WDM network. Given that wavelength converters nowadays remain very expensive, how to make effective use of a limited number of wavelength converters becomes an important issue. In this paper, we present and analyze the Sparse-Partial Wavelength Conversion (SPWC) network architecture, which has the inherent flexibility that can facilitate network carriers to upgrade the legacy optical backbone to support wavelength conversion. We explore the efficiency of partial wavelength conversion and demonstrate that SPWC architecture can significantly reduce the number of wavelength converters, yet achieve excellent blocking performance with a proper wavelength converter placement scheme. We also propose a wavelength assignment scheme called Minimum Converter Allocation (MCA), which can further improve the utilization of the wavelength converters. Simulation results indicate that, with the proposed MCA wavelength assignment algorithm, the performance of a wavelength-routed WDM network with only 1-5% of wavelength conversion capability is very close to that with Full-Complete Wavelength Conversion capability.

Keywords:
Wavelength Division Multiplexing; wavelength conversion; blocking probability

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I. INTRODUCTION AND RELATED WORK

Wavelength-routed all-optical WDM networks are considered to be candidates for the next generation wide-area backbone networks [1]. A physical wavelength-routed network consists of a set of wavelength routers (or optical cross-connects) connected by fiber links. By using the Wavelength Division Multiplexing (WDM) technique, each fiber link can support a number of wavelength channels. Wavelength routers are used to switch optical signals from an input port to an output port based on the input wavelengths. Two wavelength routers can therefore communicate with each other by setting up an all-optical "lightpath" in between, which is a direct optical connection without any intermediate electronics. In a dynamic wavelength-routed WDM network, a sequence of lightpath requests arrives over time and each lightpath holds for some period. Due to the capacity limitation of the network, some lightpath requests may not be accepted, resulting in blocking. One of the primary design objectives in dynamic wavelength-routed WDM networks is thus to minimize this blocking probability.

To establish an all-optical lightpath, it is required to allocate the same wavelength channels on all the fiber links along the path. This is known as the *wavelength continuity constraint*, which makes the wavelength-routed WDM networks different from traditional circuit-switched telephone networks. Such a constraint can be eliminated by introducing wavelength converters, which can convert the optical signals from one wavelength to another [2]. Yoo provides a comprehensive survey of wavelength conversion technologies in [3]. In this paper, a wavelength router with wavelength conversion capability is called a *wavelength-convertible router*, or WCR. However, wavelength converters remain very expensive nowadays; hence, different architectures of WCR have been proposed to reduce their cost:

Full-range Wavelength Conversion versus *Limited-range Wavelength Conversion*: Full-range wavelength conversion means that an incoming wavelength can be converted to any other outgoing wavelength. This can be achieved by employing O/E/O conversion. But an all-optical wavelength converter today only has limited-range wavelength conversion capability, where an incoming wavelength can only be converted to a small subset of the available outgoing wavelengths. Some exiting research results have shown that limited-range wavelength conversion can achieve close performance to full-range wavelength conversion [9, 16, 17]. In this paper, we will assume full-range wavelength conversion.

Complete Wavelength Conversion versus *Partial Wavelength Conversion*: Fig. 1(a) illustrates an example of WCR with Complete Wavelength Conversion capability, in which each output port is equipped with a dedicated wavelength converter. This ideal WCR is able to convert all the input wavelengths to any other wavelengths simultaneously without any limitation. Note that the number of converters is equal to the number of the fiber links multiplied by the number of wavelengths per fiber. Since the number of wavelengths could be hundreds or even more, the number of converters required will be very large and the cost of such architectures can be prohibitively high. Fortunately, it has been shown that a WCR with a limited number of converters can achieve very close performance to Complete Wavelength Conversion [4, 5]. This is referred to as Partial Wavelength Conversion. Fig. 1(b) shows the architecture of a WCR with share-per-node partial wavelength conversion [4]. A pool of wavelength converters are shared by all the output ports. This architecture requires much fewer number of wavelength converters. However, it is more complex than a wavelength router without wavelength conversion because it needs an additional small optical switch (OSW). In addition, it remains unknown how many converters should be equipped in a WCR.

Full Wavelength Conversion versus *Sparse Wavelength Conversion*: If all the wavelength routers in the network support wavelength conversion (either complete conversion or partial conversion), we call it Full Wavelength Conversion. On the other hand, if only a fraction of the wavelength routers can perform wavelength conversion, we call it Sparse Wavelength Conversion [6]. The latter has received much attention recently because it can significantly reduce the number of WCRs. It also offers a flexible solution for the network carriers to upgrade their network gradually to support wavelength conversion. To date, most of the existing studies simply assume that the WCRs in a Sparse Wavelength Conversion network all have the capability of Complete Wavelength Conversion, which however is very costly and ineffective in practice.

In this paper, we first analyze Partial Wavelength Conversion that leads to the following observations: Firstly, in order to achieve small blocking probability, over-provisioning is usually done in the backbone network. This implies that only a relatively small portion of the network capacity is used to carry the traffic to guarantee a very low blocking probability. Secondly, only the lightpaths that pass through a WCR could require allocating a wavelength converter on this WCR. Hence, from a WCR's perspective, as long as the number of ongoing bypassing lightpaths is not large, a small number of converters are enough. Thirdly, a wavelength assignment algorithm, if carefully designed, can further reduce the number of converters, and most of the lightpaths can be set up successfully without wavelength conversion.

Based on such observations, we are interested in investigating the Sparse-Partial Wavelength Conversion (SPWC) network architecture, which integrates the advantages of Partial Wavelength Conversion and Sparse Wavelength Conversion. In such networks, only a fraction of wavelength routers are WCRs with Partial Wavelength Conversion, while other wavelength routers have no wavelength conversion capability. This architecture has two important advantages: 1) it can significantly reduce the number of wavelength converters needed; 2) it is flexible for the network carrier to migrate their backbone network to support wavelength conversion, either by adding more converters into the WCRs, or by replacing the common wavelength routers with new WCRs. In our previous work [15], an approximate analytical model has been developed to evaluate the performance of SPWC architecture. However, we have not considered the impact of wavelength converter placement and wavelength assignment issues.

Though the wavelength converter placement problem has been extensively studied for the Sparse Wavelength Conversion case [7, 8, 10, 11], the corresponding problem for the SPWC case is quite different because we need to decide the number of converters for each WCR. To this end, we re-define the converter placement problem for the SPWC network architecture and propose a simple but effective solution. We further propose a novel wavelength assignment algorithm called Minimum Converter

Allocation (MCA), which can improve the utilization of wavelength converters. Our simulation results demonstrate that only 1-5% wavelength converters, if appropriately placed, are needed to achieve comparable performance to that of Full-Complete Wavelength Conversion.

The rest of the paper is organized as follows. In Section II, we present quantitative analysis on why Partial Wavelength Conversion can achieve almost the same performance as Complete Wavelength Conversion. In Section III, we describe the SPWC network architecture, and then investigate the wavelength converter placement problem. In Section IV, we present the MCA wavelength assignment algorithm which aims to use the minimum number of wavelength converters to setup a lightpath. Numerical results are presented in Section V. Finally, Section VI concludes the paper.

II. Analysis of Partial Wavelength Conversion

In this section, we analyze Partial Wavelength Conversion and show why it can achieve very good blocking performance compared to Complete Wavelength Conversion. Our key observations in this section are: (1) to guarantee a small blocking probability, the total network traffic carried in the network has to remain relatively low. Hence, the number of lightpaths concurrently passing through a wavelength router is relatively small as compared to its theoretical capacity. (2) A well-designed wavelength assignment algorithm can further decrease the number of wavelength converters. These two observations serve as the basis for the SPWC architecture.

A. Network Assumptions and notations

We first give some assumptions and notations for our network model, as follows:

1. An arbitrary mesh WDM network is represented by a graph which consists of N nodes (i.e., WCRs) and J links. The nodes are labelled from 1 to N, and the links are labelled from 1 to J.
2. The nodal degree of node n is denoted by $D(n)$, $1 \le n \le N$.
3. The number of converters inside node n is denoted by $F(n)$.

4. For simplicity, we consider bi-directional links. Each link can support W wavelengths in both directions.
5. We assume that lightpath connection requests for end-to-end node pair a follows a Poisson process with rate A_a. We also assume that the connection holding times are exponentially distributed with a unit time. The total traffic load offered to the network is T Erlangs, which are evenly distributed to all the node pairs.
6. We assume the fixed shortest path routing algorithm is used. The selected route between node pair a is denoted by R_a, and the hop-length of route R_a is $h(R_a)$. We further define that the *i*th link of route R_a is $R_a(i), 1 \le i \le h(R_a)$. We also assume the random wavelength assignment algorithm is used.

B. Link Utilization Analysis

We now show a simple model to calculate the blocking probability of a wavelength-routed WDM network with Full-Complete Wavelength Conversion. In such a network, each node is a WCR with Complete Wavelength Conversion, i.e., $F(n) = D(n)W$.

The overall blocking probability B is defined as the ratio of the blocked traffic to offered traffic. If we denote the blocking probability of route R_a by B_{R_a}, then we have

$$B = \sum_a (A_a B_{R_a}) / \sum_a A_a . \qquad (1)$$

To obtain the steady-state probability of the number of free wavelengths on each link, we use the reduced-load approximation method presented in [12]. Let X_j denote the random variable representing the number of free wavelengths on link j. We make an approximate assumption that random variables $X_j, j \in \{1, \ldots, J\}$ are independent, and the connection requests arriving at link j follow a Poisson distribution with rate α_j. Let $q_j(m_j)$ denote the probability that m_j wavelengths are free on link j.

According to our assumption, the arriving and serving behavior on the link forms an $M/M/m/m$ (m-server loss) system. Therefore we have

$$q_j(m_j) = P(X_j = m_j) = \frac{\prod_{i=1}^{m_j}(W-i+1)}{\alpha_j^{m_j}} P(X_j = 0). \quad (2)$$

and

$$q_j(0) = P(X_j = 0) = \left[1 + \sum_{m_j=1}^{W} \frac{\prod_{i=1}^{m_j}(W-i+1)}{\alpha_j^{m_j}}\right]^{-1}. \quad (3)$$

Following the approximation made in [13] for the carried traffic on link j, we can determine α_j by the following equation,

$$\alpha_j(1-q_j(0)) = \sum_{\substack{a,\text{ where link } j \\ \text{belongs to } R_a}} A_a(1-B_{R_a}). \quad (4)$$

A lightpath can be setup on a route if and only if every link on the route has free wavelengths. Thus we can calculate the blocking probability of a route according to the following equation:

$$B_{R_a} = 1 - \prod_{i=1}^{h(R_a)} (1 - q_{R_a(i)}(0)). \quad (5)$$

The above equations lead to a set of fixed-point non-linear equations, which can be solved by iterative substitutions as follows:

1. Initialize B_{R_a} to 0 for all routes, and $q_j(0)$ to 0 for all links.
2. Determine α_j using Eq. (4) for all links.
3. Determine $q_j(m_j)$ using Eqs. (2) and (3) for all links.
4. Determine B_{R_a} for all routes using Eq. (5). If the new values of B_{R_a} converge to the old values, the iteration is terminated and we can go to Step (5). Otherwise go to Step (2) for the next iteration.
5. Finally, determine the overall blocking probability B using Eq. (1).

For illustration, we use the above method to calculate the blocking probability for the 14-node NSFNET topology (Fig. 4(a)). We assume that the traffic load is uniformly distributed to all the node pairs. Suppose the average route length is denoted by L, we can simply estimate the average wavelength utilization U using $U = \frac{T*(1-B)*L}{W*J}$. According to our shortest path routing scheme, the average route length L is calculated to be 2.18. We then get the results shown in Table 1, given that the overall blocking probability B equals 2%. We can observe that the average wavelength utilization is only around 60%.

Table 1 Total traffic that can be carried on NSFNET, under a blocking probability of 2%

Number of Wavelengths: W	40	50	60	70	80	90	100
Total Traffic in Erlangs: T	208	270	333	397	460	525	590
Wavelength Utilization: U	56%	58%	59%	61%	61%	62%	63%

C. Analysis of Node Bypassing Traffic

After knowing how much traffic the network can handle, we can further analyze the traffic bypassing each node. A lightpath does not need wavelength conversion at the two end nodes; thus for each node, only the bypassing lightpaths could potentially need wavelength conversion. Hence, what we are interested in is the number of concurrent lightpaths bypassing each node, which does not count the lightpaths that are originated from or terminated at that node. Following our previous assumption, the bypassing lightpaths arriving at node n also follow a Poisson process with rate β_n:

$$\beta_n = \sum_{\substack{a, \text{ where route } R_a \\ \text{bypasses node } n}} A_a \,. \tag{6}$$

Let $p_n(f_n)$ denote the probability that f_n lightpaths are concurrently bypassing node n. So f_n varies from 0 to $F(n)$ because node n can support at most $F(n)$ lightpaths. According to our assumption, the arrival and departure behavior of the bypassing lightpaths on each node also forms an $M/M/m/m$ system. So we have

$$p_n(f_n) = \frac{(\beta_n)^{f_n} \frac{1}{f_n!}}{\sum_{i=0}^{F(n)} (\beta_n)^i \frac{1}{i!}}, \quad 0 \le f_n \le F(n). \tag{7}$$

Given that the blocking probability of the $M/M/m/m$ system is very low, the $M/M/m/m$ system can be approximated by an $M/M/\infty$ system. Hence, the average number of bypassing lightpaths can be simply approximated by β_n, because the service time is assumed to be exponentially distributed with unit time. We still use NSFNET as an example. Assume each link can support 40 wavelengths, and the blocking probability is 2%. From the results of Section II-B, the total traffic is 208 Erlangs and each node pair has a traffic of 2.286 Erlangs. We can therefore calculate the bypassing traffic for each node using Eq. (6). The results are shown in Table 2.

Table 2 β_n and $F(n)$ of NSFNET when total traffic = 208 Erlangs

Node n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
β_n	11.4	18.3	11.4	45.7	11.4	27.4	25.1	2.3	18.3	36.6	16.0	18.3	0	4.6
$F(n)$	120	120	120	160	80	120	120	80	120	160	120	120	80	80

We find that the ratio of $\beta_n / F(n)$ ranges from 0 to 28.6%, and in most cases it is only about 10-15%. This implies that the number of lightpaths concurrently bypassing a node is very small compared to the node's full capacity. We also show the curve of $p_n(f_n)$ in Fig. 2 for nodes 1 and 4, which plots the probability distribution of the number of bypassing lightpaths. We can see that the probability that more than 20 lightpaths are concurrently bypassing node 1 is almost zero. Therefore, for node 1, 20 wavelength converters are enough to achieve almost the same effect as Complete Wavelength Conversion which requires 120 converters. For node 4, which has the highest volume of bypassing traffic, the probability that more than 70 lightpaths are concurrently bypassing it is almost zero.

One reason for the small volume of bypassing traffic in NSFNET is that the average route hop length is only 2.18. For a network with an average route length of *L*, the percentage of bypassing traffic should be approximately $\frac{L-1}{L+1}$ because in a route with length *L*, the lightpath will pass through *L*+1 nodes and only the intermediate *L*-1 nodes consider the lightpath as a "bypassing" one. That is to say, for each node, about $\frac{2}{L+1}$ of the lightpaths are not bypassing lightpaths. We argue that, for most optical backbone networks with modest network size and network diameter, the average route lengths are not expected to be very large; otherwise the efficiency of the network will be very low [3].

To conclude, from a node's perspective, a considerable percentage of lightpaths is not bypassing. Since only the bypassing lightpaths require wavelength conversion, it is possible to equip a small number of wavelength converters in each node to achieve satisfactory performance.

D. Wavelength Assignment

In the previous subsection, we conclude that only a limited number of lightpaths can bypass a node concurrently. In this section, we further show that for these bypassing lightpaths, most of them do not need wavelength conversion if an appropriate wavelength assignment is employed.

We conduct simulations for the NSFNET topology without wavelength conversion, i.e., each lightpath has to use the same wavelength on all its links. The total network traffic is 208 Erlangs. We use the First-fit wavelength assignment scheme in our simulation [14]. For each node, we get the percentage of the bypassing lightpaths that are set up successfully, as shown in Table 3. We observe that more than 90% of the bypassing lightpaths can be set up without wavelength conversion by using the simple First-fit wavelength assignment scheme. In other words, no more than 10% of the bypassing lightpaths actually need wavelength conversion. Recall the results of Section II-C that the number of concurrently bypassing lightpaths on node *n* are much less than the value of $D(n)W$, we can therefore conclude that a very small number of wavelength converters can achieve almost the same performance as complete wavelength conversion.

Table 3 The percentage of the bypassing lightpaths that are setup successfully

Node *n*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
Per.	96.9%	95.3%	97.8%	93.9%	96.0%	92.9%	92.3%	100%	95.0%	95.5%	94.8%	96.0%	100%	97.5%

III. SPARSE-PARTIAL WAVELENGTH CONVERSION

In this section, we first introduce the SPWC network architecture based on the observations made in the previous section. We then discuss the wavelength converter placement issue.

A. *Sparse-Partial Wavelength Conversion*

Given that wavelength conversion technology remains immature, it is not practical for the network carrier to replace all the wavelength routers by WCRs. Since it has been shown that Sparse Wavelength Conversion can achieve very close performance to Full Wavelength Conversion, we combine the advantages of Partial Wavelength Conversion and Sparse Wavelength Conversion to form the SPWC network architecture. There are two kinds of nodes in such network: nodes without wavelength conversion capability, and nodes with Partial Wavelength Conversion capability. By using Sparse Wavelength Conversion and Partial Wavelength Conversion together, only a small number of wavelength converters are needed to achieve comparable performance to Full-Complete Wavelength Conversion. As such, it only requires that a small fraction of wavelength routers be replaced with WCRs, which is very flexible for the network carriers to migrate the existing network to support wavelength conversion.

A SPWC network operates as follows: Upon the arrival of a lightpath request, if there exists any link in the selected route which currently has no free wavelength, we cannot set up the lightpath on this route. Otherwise, we should first try to find a common free wavelength on all the links along the selected path. We have shown in Section II-D that most of the lightpath requests can be set up in this way without using any wavelength converters. If there is no common free wavelength, we then check whether wavelength converters can help. A lightpath is divided into several segments by all the intermediate WCRs having free converters, as shown in Fig. 3. Notice that a WCR may have no conversion capability if all its wavelength converters have been allocated. Each segment still suffers the wavelength continuity. The lightpath can be setup successfully if and only if every segment has common free wavelength(s). Hence we have to check whether there exist common free wavelengths for each segment individually. Wavelength converters will be allocated accordingly if necessary. Once the lightpath is terminated, the allocated converters will also be released.

B. Wavelength Converter Placement Problem

A very important problem in the SPWC network architecture is the placement of wavelength converters. Traditionally, this problem is defined in the context of Sparse Wavelength Conversion, that is, to determine a set of routing nodes with Complete Wavelength Conversion capability such that the overall network blocking probability can be minimized. In our SPWC network architecture, we should re-define the wavelength converter placement problem as two sub-problems: (1) how to find a set of nodes which will be placed with a WCR? (2) Given the total number of M converters, how to place them in the selected WCRs? We now propose a simulation-based scheme to solve these two sub-problems. Its performance is evaluated in Section V.

The basic idea of our scheme is to conduct simulations assuming Full-Complete Wavelength Conversion; from the simulations, we can observe how many wavelength conversions are conducted in each node. Thus we obtain statistics of the average number of busy converters for each node n, denoted by $A(n)$. It is very intuitive to place more wavelength converters on the nodes with large values of $A(n)$. In the following, we use the NSFNET topology as an example to show how to choose the WCRs and assign wavelength converters to each WCR.

In this example, we assume each fiber link can support 40 wavelengths. In the simulations, 1,000,000 consecutive lightpath requests are generated. The total network traffic is 200 Erlangs and they are uniformly distributed to all the node pairs. The following routing and wavelength assignment algorithm is used to setup a lightpath:

Upon the arrival of a lightpath request:

1. Find the shortest path between the two end nodes of the lightpath request.
2. In the shortest path, if there exists any link that has no free wavelength, the lightpath request will be blocked.
3. If there exist common free wavelengths among all the links in the route, set up the lightpath by choosing the common free wavelength with the smallest label for each link.
4. Otherwise, for each link, use the first-fit wavelength assignment scheme. If two consecutive links use different wavelengths, a wavelength converter is allocated in the intermediate WCR.

We call the above wavelength assignment scheme in Steps 3 and 4 Modified First-Fit (MFF) wavelength assignment. Step 3 is particularly important as we have shown in Section II-D that 90% of the bypassing lightpaths can be set up without wavelength conversion under low traffic.

Our wavelength converter placement algorithm works as follows:

INPUT:

$\{A(n)\}$, $1 \le n \le N$, and an integer M which represents the number of converters.

CONVERTER_PLACEMENT ($\{A(n)\}$**,** M)

$C = \phi$;

Calculate the mean value of $A(n)$ as $\bar{A}$;

Calculate the standard deviation of $A(n)$ as S;

for($n = 1$; $n \le N$; n++)

    if ($A(n) \ge \bar{A} + 0.8S$)

        $C = C \bigcup \{n\}$;

if ($|C| \geq M$) /* $|C|$ is the cardinality of set C */
    Find a set *D* such that $D \subseteq C$ and $|D| = M$ and $\forall x \in D, \forall y \notin D, A(x) > A(y)$;
    for $\forall n \in D$
        Place one converter at node *n*;
    return;
else
    Sort the elements in set *C* as $r_1, r_2, ..., r_{|C|}$, such that $\forall i > j, A(r_i) \geq A(r_j)$;
    $B[0] = 0$;
    for ($n = 1$; $n \leq |C|$; n++)
        $M = M - B[n-1]$;
        $B[n] = (\text{int}) \frac{A(n)}{\sum_{i=n}^{|C|} A(i)} M$;
        Place $B[n]$ number of converters at node r_n;
    return;

In the above example, the simulation results of the $A(n)$ values of node 1 to 14 are {0.4, 0.7, 0.3, 2.3, 0.4, 1.8, 1.6, 0, 0.7, 1.4, 0.7, 0.6, 0, 0.1}. We can observe that the utilization of wavelength converters is indeed very low. Based on our converter placement algorithm, given 50 converters, we will assign the converters as follows: {Node 4: 16, Node 6: 13, Node 7: 11, Node 10: 10}.

IV. MINIMUM CONVERTER ALLOCATION WAVELENGTH ASSIGNMENT ALGORITHM

In the previous section, we have shown that the SPWC network architecture can significantly reduce the number of wavelength converters, even by using a simple MFF wavelength assignment algorithm. In this section, we propose a wavelength assignment algorithm specifically designed for SPWC which can further improve the utilization of wavelength converters.

In a lightpath, each segment has to use the same wavelength on its links. Hence, a lightpath can be setup on a route successfully if and only if every segment has at least one common free wavelength. In the following, we assume that the selected route has been checked and every segment has at least one common free wavelength.

In the MFF wavelength assignment algorithm, each segment chooses the free wavelength with the smallest label individually. However, we know that if two consecutive segments happen to use the same wavelength, then the intermediate WCR does not need to allocate a wavelength converter. Therefore, to minimize the number of allocated wavelength converters, it is necessary to optimally assign wavelengths to the segments. The rationale is to maximize the number of cases that two consecutive segments use the same wavelength. This optimization problem can be formulated as follows:

Given:

1) A set of wavelengths on every link, labeled as $\{1, 2, \cdots, W\}$;

2) A path composed by K segments: $\{1, 2, \cdots, K\}$;

3) A set of free wavelengths on segment i, denoted by S_i, $S_i \subseteq \{1, 2, \cdots, W\}$ and $S_i \neq \Phi$ (Otherwise, the lightpath will be blocked on this route.)

4) $\bigcap_{i=1}^{K} S_i = \Phi$. (Otherwise, we can setup the lightpath without wavelength converters.)

Problem:

Find $\lambda_1 \in S_1, \lambda_2 \in S_2, \ldots, \lambda_K \in S_K$ so as to maximize C, which is defined as: $C = \sum_{i=1}^{K-1} c_i$ where $c_i = \begin{cases} 1 & \lambda_i = \lambda_{i+1} \\ 0 & \lambda_i \neq \lambda_{i+1} \end{cases}$.

The above problem can be solved by a dynamic programming solution shown as follows:

Solution:

Consider the sub-problem that involves segments 1 through k only, and assume the assigned wavelength on segment k is λ_m. We use $F(k,m)$ to represent the maximum of C for this sub-problem. Then the solution to the original problem is $\max_{m \in S_K} F(K,m)$.

Now let us derive a recurrence relation for $F(k,m)$.

1) $k = 1$, $m \in S_1$: $F(1,m) = 0$ if $m \in S_1$;

2) $1 < k \leq K$, $m \in S_k$: $F(k,m) = \max\left\{ F(k-1,m)+1, \max_{n \in S_{k-1}, n \neq m} F(k-1,n) \right\}$;

3) otherwise, $F(k,m) = -\infty$. □

The time to fill F is bounded by $O(KW^2)$. The corresponding optimal wavelength assignment can be found by backtracking the recurrence relation.

To summarize, the rationale of our wavelength assignment algorithm is to minimize the number of allocated wavelength converters for each lightpath request. Hence, we call it the Minimum Converter Allocation (MCA) wavelength assignment algorithm, and it works as follows:

After a route is chosen:

If there exist common free wavelengths among all the links in the route, set up the lightpath by choosing the common free wavelength with the smallest label for each link. In this case, no wavelength converter is allocated.

Otherwise, assign wavelengths to each segment using the dynamic programming approach, i.e., assign wavelength λ_i to segment S_i.

The performance of the MCA wavelength assignment algorithm will be presented in Section V-B.

V. NUMERICAL RESULTS AND ANALYSIS

In this section, we compare the performance of different conversion schemes for NSFNET topology and 25-node mesh-torus network topology (Fig. 4). As in many previous studies, we assume that the traffic is uniformly distributed to all node pairs. The lightpath requests arrive according to a Poisson process and the holding time is exponentially distributed with a unit time. We assume 40 wavelength channels are available for each fiber link. In our simulations, every single data is obtained by conducting 30 independent replications of the same simulation and then calculating the mean results. The confidence level of the simulations is 95% and the relative error is within 5%.

A. Performance Evaluation of Sparse-Partial Wavelength Conversion

In this set of experiments, we mainly focus on the efficiency of sparse-partial wavelength conversion. We compare the blocking performance of sparse-partial conversion to that of no conversion and full-complete conversion.

Fig. 5 shows the blocking probabilities of the total network versus traffic load for different wavelength conversion schemes in the NSFNET topology. In the simulations, we use the Shortest Path Routing and First-Fit wavelength assignment algorithm for the case with no wavelength conversion, and the Shortest Path Routing and MCA wavelength assignment algorithm for SPWC. The first observation from the figure is that full-complete wavelength conversion can decrease the blocking probability by a large margin. The second significant result is that, compared to the 1,600 converters used in the full-complete wavelength conversion, only 50 converters can achieve satisfactory performance if sparse-partial wavelength conversion schemes are used.

Fig. 6 plots the blocking performance of the 25-node mesh-torus network. In this topology, full-complete wavelength conversion requires 25 WCRs and 4,000 wavelength converters. Given 75 wavelength converters, our converter placement algorithm places 15 converters at WCR 1, 2, 3, 4, 5, respectively. We then conduct simulations for different conversion cases: no conversion, sparse-partial wavelength conversion with 5 WCRs, where each WCR has 15 wavelength converters, and the full-complete wavelength conversion. The analytical results for sparse-partial wavelength conversion are also presented. We can observe that SPWC works very well in the mesh-torus topology: 5 WCRs with a total of 75 wavelength converters can achieve almost the same performance as 4,000 wavelength converters. To show the advantage of our converter placement algorithm, we also plot the performance of another simple converter placement scheme which places the converters to all the nodes evenly. Give 75 converters, each node will have 3 converters. One immediate disadvantage of this scheme is that all the nodes should be able to support wavelength converters. In terms of performance, we can see that our converter placement scheme can achieve much lower blocking probabilities.

B. Performance of the MCA Wavelength Assignment Algorithm

In this set of experiments, we will show the advantage of the MCA wavelength assignment algorithm proposed in Section IV.

Fig. 7 shows the blocking probabilities of the 14-node NSFNET, with a fixed total traffic load of 210 Erlangs, for different numbers of wavelength converters and different wavelength assignment algorithms. The top curve in Fig. 7 shows the performance of the simple First-Fit wavelength assignment algorithm, which assign wavelengths to each segment individually without checking the whole route. Although the performance can be improved by adding more wavelength converters, it is inefficient and requires lots of wavelength converters. The MFF wavelength assignment algorithm works much better. We can see that, after the number of wavelength converters increases to 70, the blocking performance has reached the optimum value and cannot be decreased any more. On the other hand, the MCA wavelength assignment algorithm can approach that performance bound by using only 50 wavelength converters. This is a significant reduction as compared with the 70 wavelength converters required by the MFF wavelength assignment algorithm.

Similar results are observed in Fig. 8, which plots the blocking probabilities in the 25-node mesh-torus network with a fixed total traffic load of 400 Erlangs. The MFF wavelength assignment algorithm requires 100 wavelength converters to achieve the best blocking probability, while the MCA wavelength assignment algorithm required only 75 wavelength converters, which means 25% cost reduction.

VI. CONCLUSIONS

This paper addresses an important problem in wavelength-routed all-optical WDM networks: how to efficiently utilize a limited number of wavelength converters. We first explain why Partial Wavelength Conversion can achieve very close performance to Complete Wavelength Conversion. We then study the Sparse-Partial Wavelength Conversion network architecture, which has the flexibility to install the partial WCRs gradually into the network. We further investigate the wavelength converter placement and wavelength assignment issues. We have shown that by using the proposed wavelength converter placement scheme and wavelength assignment algorithm, a very small number of wavelength converters can achieve very close performance to that of the Full-Complete Wavelength Conversion.

There are many possible future research directions within this framework. For example, in this paper, we assume that static shortest path routing is used. Given that many adaptive routing algorithms are effective in reducing blocking probability, it is possible to use them in our SPWC architecture. Wavelength converter placement and wavelength assignment under such advanced routing algorithms are also interesting issues worthy of further investigation.

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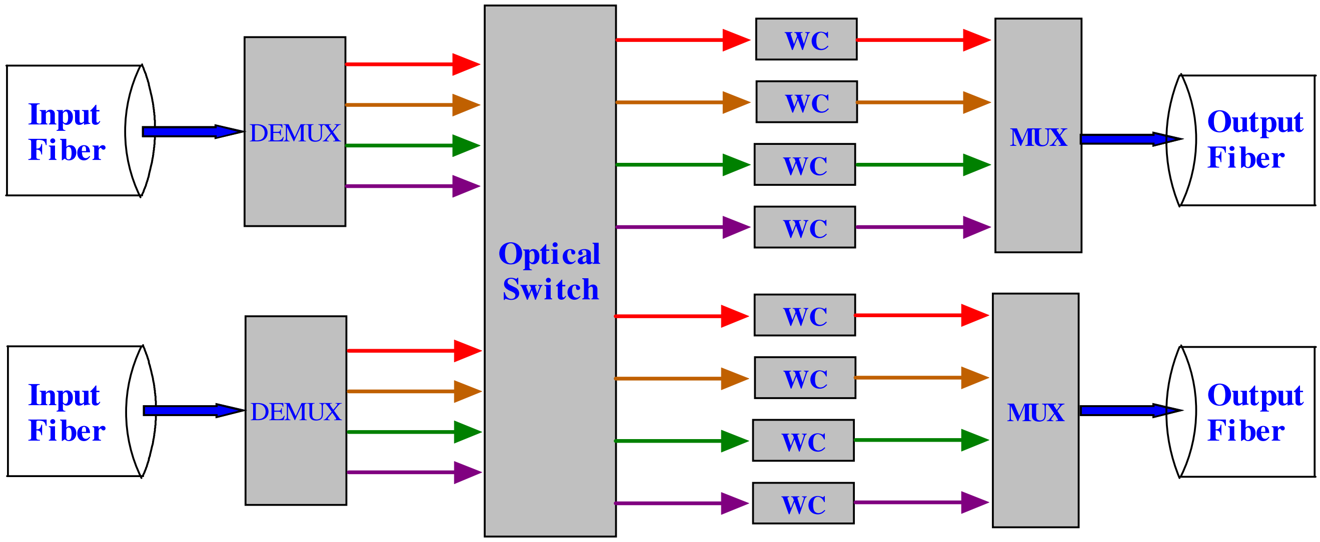
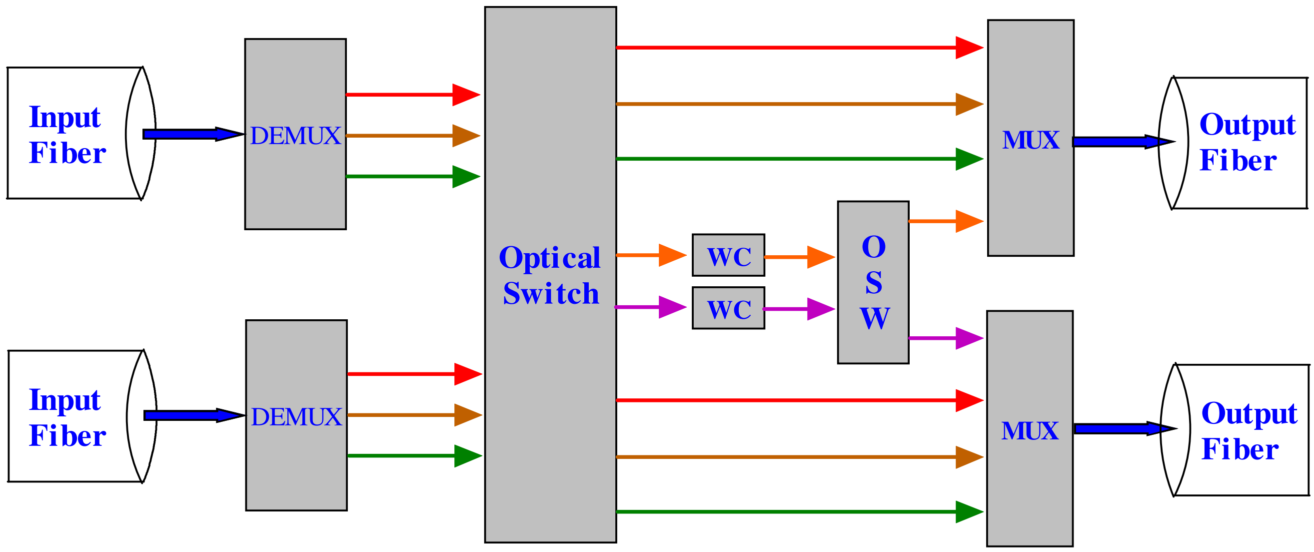
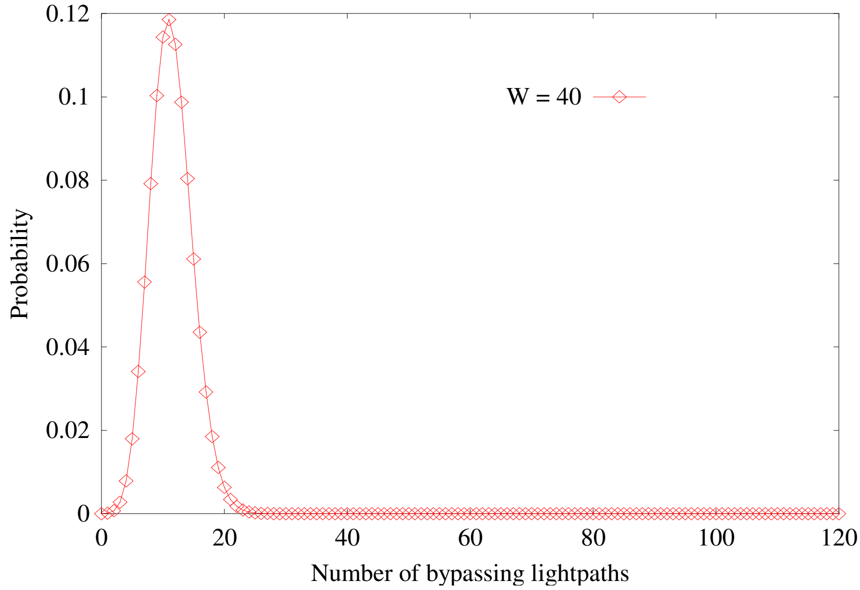


(a) A WCR with full wavelength conversion

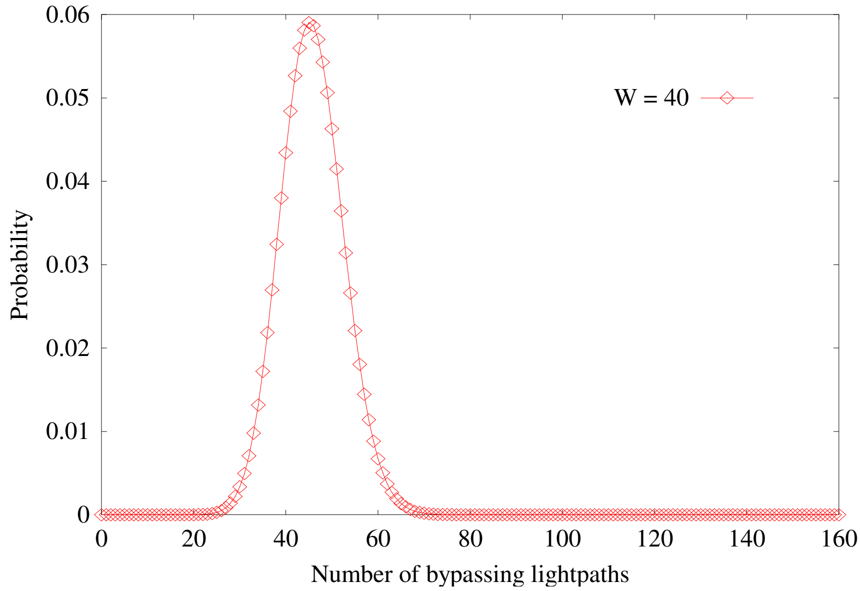


(b) A WCR with partial wavelength conversion

Fig. 1. WCR architecuters



(a) Node 1, $F(n) = 120$



(b) Node 4, $F(n) = 160$

Fig. 2. Probability distribution of the number of bypassing lightpaths for Node 1, $F(n) = 120$

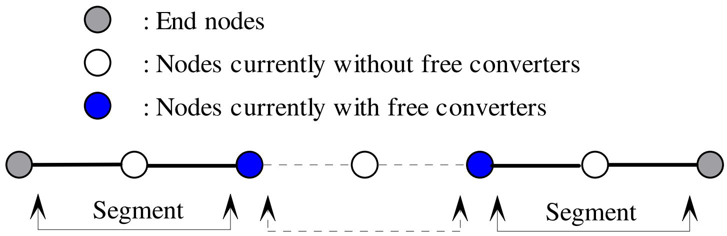
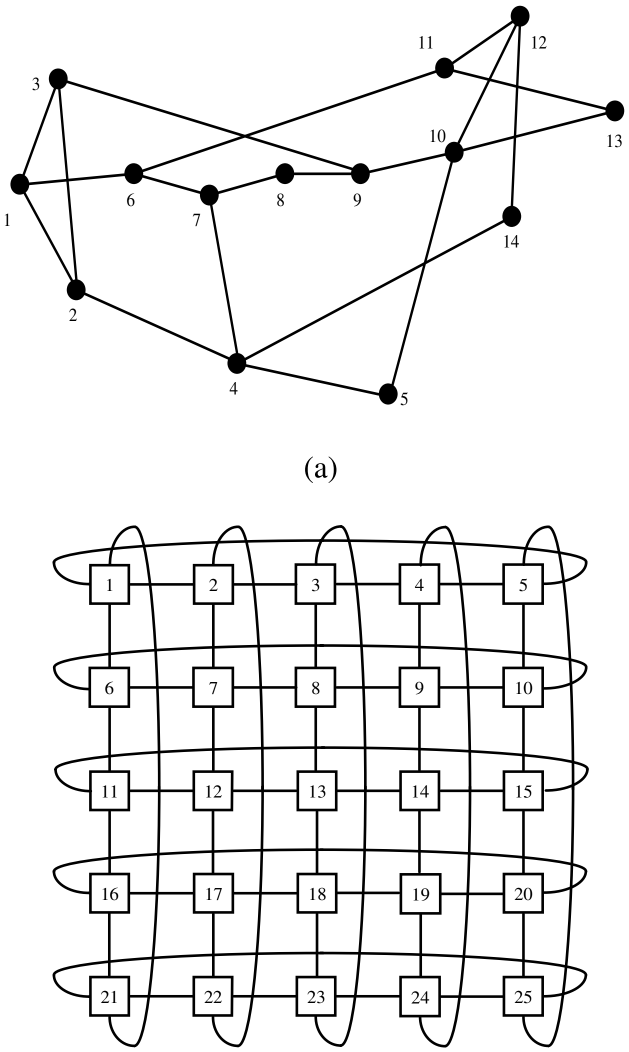


Fig. 3. A lightpath and its segments



(b)

Fig. 4. Network Topologies: (a) 14-node NSFNET (b) 25-node Mesh-torus

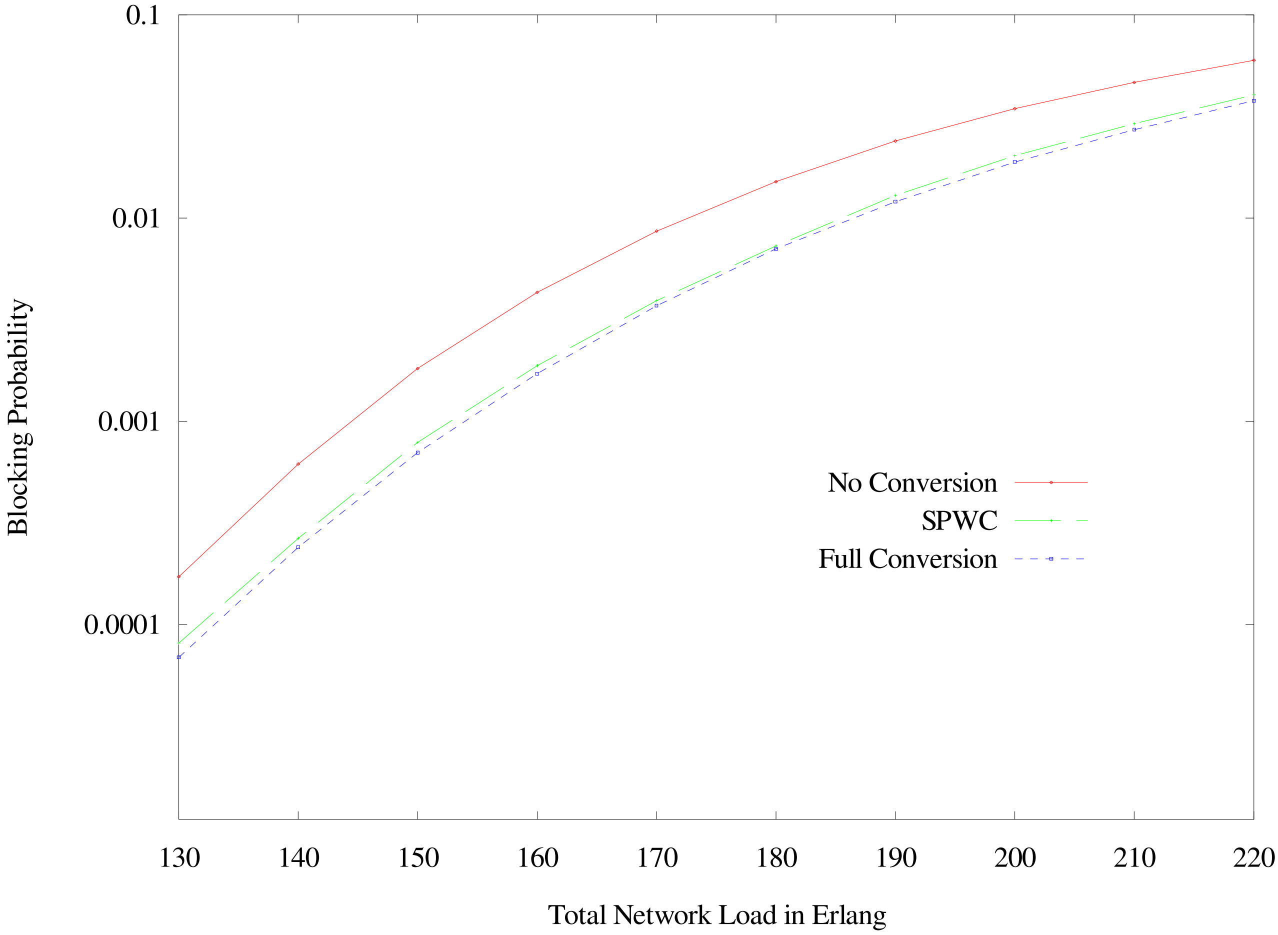


Fig. 5. Blocking Performance in NSFNET, $M = 50$

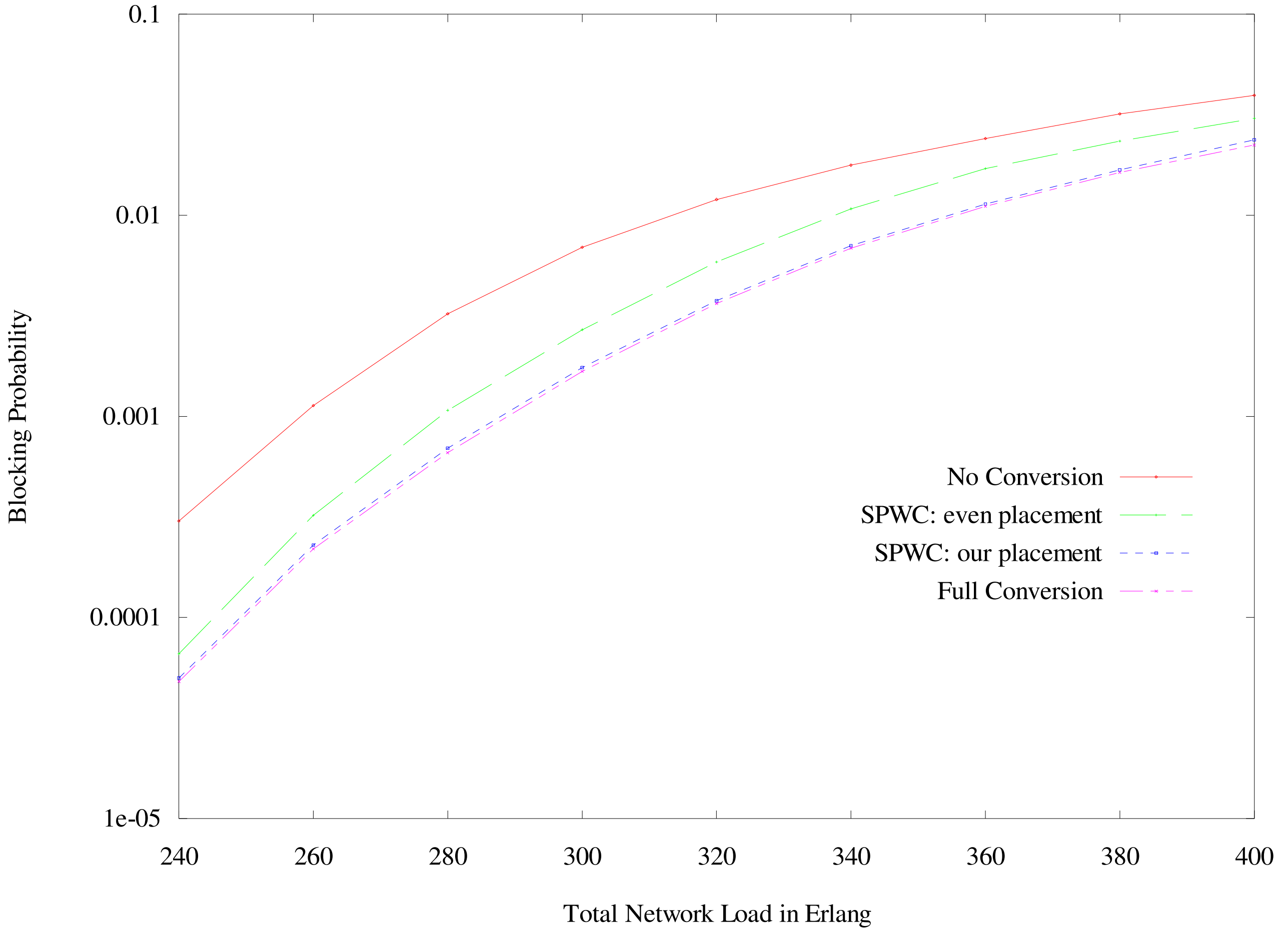


Fig. 6. Blocking Performance in 25-node mesh-torus network, $M = 75$

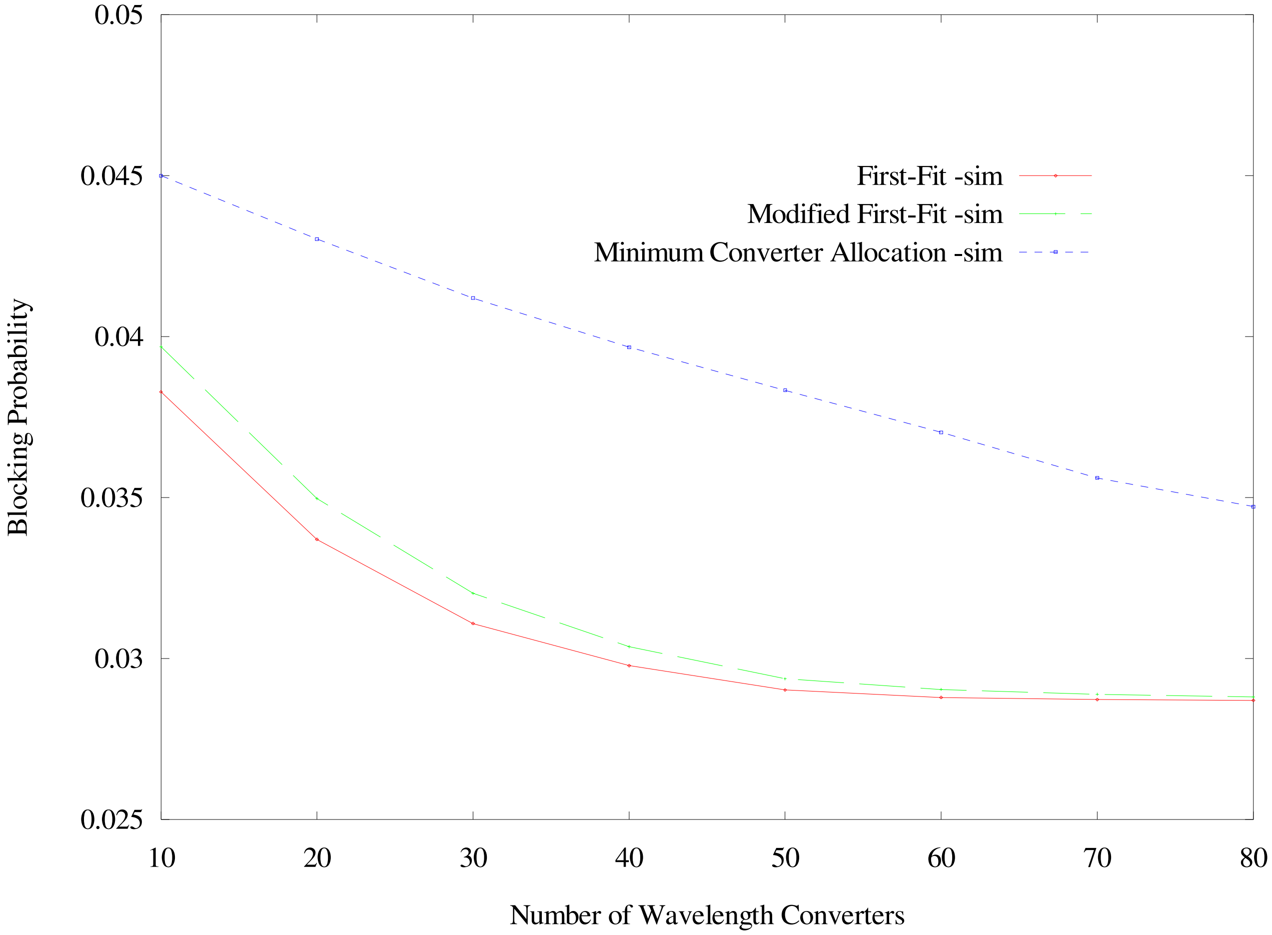


Fig. 7. Blocking Performance of different wavelength assignment algorithms in NSFNET, Total network load = 210 Erlangs

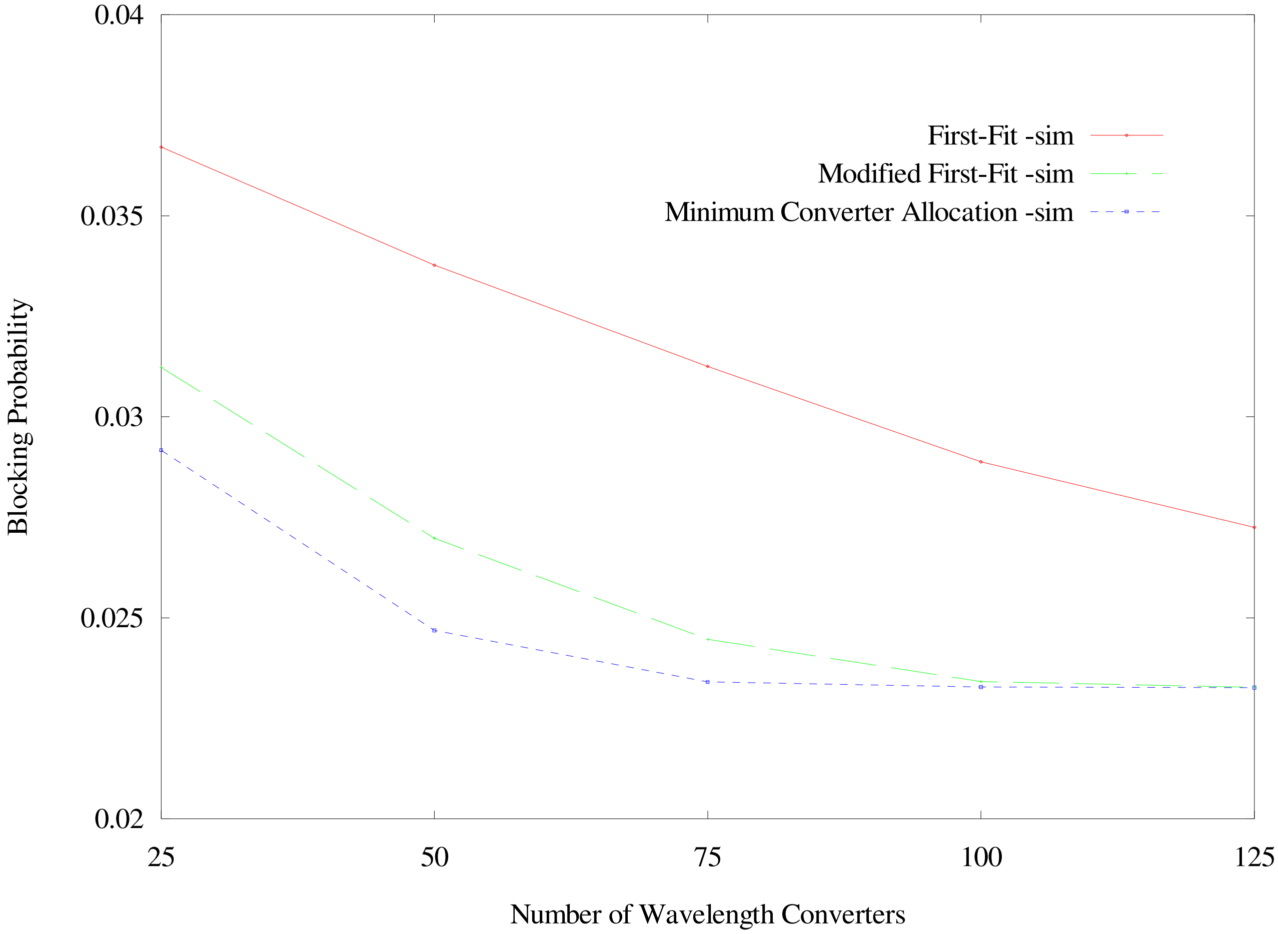


Fig. 8. Blocking Performance of different wavelength assignment algorithms in 25-node mesh-torus network, Total network load = 400 Erlangs