Diffusion Mechanism Design in Social Networks

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***Abstract*—In this article, we introduce the diffusion mechanisms that we have proposed [1], [2]. We consider a market where a seller sells multiple units of a commodity in a social network. Each node/buyer in the social network can only directly communicate with her neighbours, i.e. the seller can only sell the commodity to her neighbours if she could not find a way to inform other buyers. We have designed a novel promotion mechanism that incentivizes all buyers, who are aware of the sale, to invite all their neighbours to join the sale, even though there is no guarantee that their efforts will be paid. While traditional sale promotions such as sponsored search auctions cannot guarantee a positive return for the advertiser (the seller), our mechanism guarantees that the seller's revenue is better than not using our promotion mechanism. More importantly, the seller does not need to pay if the promotion is not beneficial to her. In this article, we briefly introduce our mechanism in a simple setting and highlight some open problems for further investigations.**

***Index Terms*—Mechanism design, information diffusion, revenue maximisation, algorithmic game theory**

I. Introduction

MARKETING is one of the key operations for a service or product to survive. To do that, companies often use newspapers, tv, social media, search engines to do advertisements. Indeed, most of the revenue of social media and search engines comes from paid advertisements. According to Statista, Google's ad revenue amounted to almost 79.4 billion US dollars in 2016. However, whether all the advertisers actually benefit from their advertisements is not clear and is difficult to monitor. Although most search engines use market mechanims like generalised second price auctions to allocate advertisements and only charge the advertisers when users click their ads, not all clicks lead to a purchase [3], [4]. That said, the advertisers may pay user clicks that have no value to them.

In order to guarantee that a seller never loses from using advertising, we have proposed novel advertising mechanisms without using third-party advertising platforms for the seller (to sell services or products) that do not charge the seller unless the advertising brings revenue-increase for the seller [1], [2], [5]. We model all potential buyers of a service/product as a large social network where each buyer is linked with some other buyers (known as neighbours). The seller is also located somewhere in the social network. Before the seller finds a way to inform more buyers about her sale, she can only sell her products to her neighbours. In order to attract more buyers to increase her revenue, the seller may pay to advertise the sale via newspapers, social media, search engines etc. to reach/inform more potential buyers in the social network. However, if the advertisements do not bring any valuable buyers, the seller loses the investment on the advertisements.

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Our advertising mechanism does not rely on any third party such as newspapers or search engines to do the advertisements. The mechanism is owned by the seller. The seller just needs to invite all her neighbours to join the sale, then her neighbours will further invite their neighbours and so on. In the end, all buyers in the social network will be invited to participate in the sale. Moreover, all buyers are not paid in advance for their invitations and they may not get paid if their invitations are not beneficial to the seller. Although some buyers may never get paid for their efforts in the advertising, they are still incentivized to do so, which is one of the key features of our advertising mechanism. This significantly differs from existing advertising mechanisms used on the Internet.

More importantly, our advertising mechanism not only incentivizes all buyers to do the advertising, but also guarantees that the seller's revenue increases. That is, her revenue is never worse than the revenue she can get if she only sells the items to her neighbours.

Maximising the seller's revenue has been well studied in the literature, but the existing models assumed that the buyers are all known to the seller and the aim is to maximize the revenue among the fixed number of buyers. Given the number of buyers is fixed, if we have some prior information about their valuations, Myerson [6] proposed a mechanism by adding a reserve price to the original Vickrey-Clarke-Groves (VCG) mechanism. Myerson's mechanism maximises the seller's revenue, but requires the distributions of buyers' valuations to compute the reserve price. Without any prior information about the buyers' valuations, we cannot design a mechanism that can maximise the revenue in all settings (see Chapter 13 of [7] for a detailed survey). Goldberg et al. [8], [9] have considered how to optimize the revenue for selling multiple homogeneous items such as digital goods like software (unlimited supply). Especially, the seller can choose to sell less with a higher price to gain more.

In terms of incentivizing people to share information (like buyers inviting their neighbours), there also exists a growing body of work [10], [11], [12], [13]. Their settings are essentially different from ours however. They considered either how information is propagated in a social network or how to design reward mechanisms to incentivize people to invite more people to accomplish a challenge together. The mechanism designed by the MIT team under the DARPA Network Challenge (2009) is a nice example, where they designed a novel reward mechanism to share the award if they win the challenge. Thier mechanism attracted many people via social network to join the team, which eventually helped them to win the challenge [12].

II. The Model

We consider a seller s sells $\mathcal{K} \geq 1$ items in a social network. In addition to the seller, the social network consists of n nodes denoted by $N = \{1, \cdots, n\}$, and each node $i \in N \cup \{s\}$ has a set of neighbours denoted by $r_i \subseteq N \cup \{s\}$. Each $i \in N$ is a buyer of the $\mathcal{K}$ items.

For simplicity, we assume that the $\mathcal{K}$ items are homogeneous and each buyer $i \in N$ requires at most one unit of the item and has a valuation $v_i \geq 0$ for one or more units.

Without any advertising, seller s can only sell to her neighbours r_s as she is not aware of the rest of the network and the other buyers also do not know the seller s. In order to maximize s's profit, it would be better if all buyers in the network could join the sale.

Traditionally, the seller may pay some of her neighbours to advertise the sale to their neighbours, but the neighbours may not bring any valuable buyers and cost the seller money for the advertisement. Therefore, our goal here is to design a kind of cost-free advertising mechanism such that all buyers who are aware of the sale are incentivized to invite all their neighbours to join the sale with no guarantee that their efforts will be paid.

Let us first formally describe the model. Let $\theta_i = (v_i, r_i)$ be the *type* of buyer $i \in N$, $\theta = (\theta_1, \cdots, \theta_n)$ be the type profile of all buyers and θ_{-i} be the type profile of all buyers except i. θ can also be represented by (θ_i, θ_{-i}). Let Θ_i be the type space of buyer i and Θ be the type profile space of all buyers.

The advertising mechanism consists of an *allocation policy* π and a *payment policy* x. The mechanism requires each buyer who is aware of the sale to report her valuation to the mechanism and invite all her neighbours to join the sale. Let v'_i be the valuation report of buyer i and $r'_i \subseteq r_i$ be the neighbours i has invited. Let $\theta'_i = (v'_i, r'_i)$ and $\theta' = (\theta'_1, \cdots, \theta'_n)$, where $\theta'_j = nil$ if j has never been invited by any of her neighbours r_j or j does not want to participate. Given the action profile θ' of all buyers, $\pi_i(\theta') \in \{0, 1\}$, 1 means that i receives one item, while 0 means i does not receive any item. $x_i(\theta') \in \mathbb{R}$ is the payment that i pays to the mechanism, $x_i(\theta') < 0$ means that i receives $|x_i(\theta')|$ from the mechanism.

Different from the traditional mechanism design settings, in this model, we want to incentivize buyers to not only just report their valuations truthfully, but also invite all their neighbours to join the sale/auction (the advertising part). Therefore, we extend the definition of incentive compatibility to cover the invitation of their neighbours. Specifically, a mechanism is incentive compatible (or truthful) if for all buyers who are invited by at least one of their neighbours, reporting their valuations truthfully to the mechanism and further inviting all their neighbours to join the sale is a dominant strategy.

III. The Diffusion Mechanism

In this section, we review the diffusion mechanism proposed by Zhao et al. [2] for the case of $\mathcal{K} = 1$. The essence of our mechanism is that a buyer is rewarded for advertising the sale only if her invitations increase social welfare, and the reward guarantees that inviting all neighbours is a dominant strategy for all buyers.

The diffusion mechanism is outlined below:

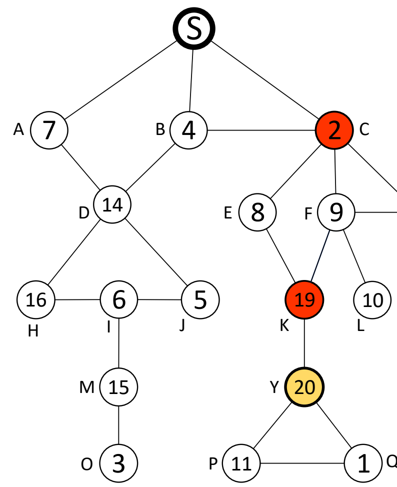

Fig. 1. A running example of the information diffusion mechanism, where the seller s is located at the top of the graph and is selling one item, the value in each node is the node's private valuation for receiving the item, and the lines between nodes represent neighbourhood relationship. Node Y is the node with the highest valuation and C, K are Y's diffusion critical buyers.

Information Diffusion Mechanism (IDM)

1) Given a feasible action profile θ', identify the buyer with the highest valuation, denoted by i^*.
2) Find all *diffusion critical buyers* of i^*, denoted by C_{i^*}. $j \in C_{i^*}$ if and only if without j's action θ'_j, there is no invitation chain from the seller s to i^* following θ'_{-j}, i.e. i^* is not able to join the sale without j.
3) For any two buyers $i, j \in C_{i^*} \cup \{i^*\}$, define an order $\succ_{i^*}$ such that $i \succ_{i^*} j$ if and only if all invitation chains from s to j contain i.
4) For each $i \in C_{i^*} \cup \{i^*\}$, if i receives the item, the payment of i is the highest valuation report without i's participation. Formally, let N_{-i} be the set of buyers each of whom has an invitation chain from s following θ'_{-i}, i's payment to receive the item is $p_i = max_{j \in N_{-i} \wedge \theta'_j \neq nil} v'_j$.
5) The seller initially gives the item to the buyer i ranked first in $C_{i^*} \cup \{i^*\}$, let $l = 1$ and repeat the following until the item is allocated.
   - if i is the last ranked buyer in $C_{i^*} \cup \{i^*\}$, then i receives the item and her payment is $x_i(\theta') = p_i$;
   - else if $v'_i = p_j$, where j is the $(l+1)$-th ranked buyer in $C_{i^*} \cup \{i^*\}$, then i receives the item and her payment is $x_i(\theta') = p_i$;
   - otherwise, i passes the item to buyer j and i's payment is $x_i(\theta') = p_i - p_j$, where j is the $(l+1)$-th ranked buyer in $C_{i^*} \cup \{i^*\}$. Set $i = j$ and $l = l + 1$.
6) The payments of all the rest buyers are zero.

Figure 1 shows a social network example. Without any advertising, the seller can only sell the item among nodes A, B and C, and her revenue cannot be more than 7. If A, B and C invite their neighbours, these neighbours further invite their neighbours and so on, then all nodes in the social network will be able to join the sale and the seller may receive a revenue as high as the highest valuation of the social network which is 20.

Let us run IDM on the social network given in Figure 1. Assume that all buyers report their valuations truthfully and invite all their neighbours, IDM runs as follows:

- Step (1) identifies that the buyer with the highest valuation is Y, i.e. $i^* = Y$.
- Step (2) computes $C_{i^*} = \{C, K\}$.
- Step (3) gives the order of $C_{i^*} \cup \{i^*\}$ as $C \succ_{i^*} K \succ_{i^*} i^*$.
- Step (4) defines the payments p_i for all nodes in $C_{i^*} \cup \{i^*\}$, which are $p_C = 16$, $p_K = 17$ and $p_Y = 19$, the highest valuation without C, K and Y's participation respectively.
- Step (5) first gives the item to node C; C is not the last ranked buyer in $C_{i^*} \cup \{i^*\}$ and $v_C \neq p_K$, so C passes the item to K and her payment is $p_C - p_K = -1$; K is not the last ranked buyer, but $v_K = p_Y$, therefore K receives the item and pays p_K.
- All the rest of the buyers, including Y, pay nothing.

In the above example, IDM allocates the item to node K and K pays 17, but s does not receive all the payment, and she pays C an amount of 1 for the advertising. Therefore, the seller receives a revenue of 16 from IDM, which is more than two times the revenue she can get without any advertising. Note that only buyer C is rewarded for the information propagation as the other buyers are not critical for inviting K.

A. *Properties of the Diffusion Mechanism*

Firstly, we can show that for all buyers who are invited by at least one of their neighbours, reporting their valuations truthfully to the mechanism (i.e. the seller) and further inviting all their neighbours to join the sale is a dominant strategy. Secondly, all buyers' utilities are non-negative, i.e. they are not forced to join the sale. Lastly, the seller's revenue is greater than or equal to the revenue she could get under the second price auction (Vickrey auction) among her neighbours only. All the properties together solve the dilemma that the seller has faced with the traditional advertising platforms such as search engines.

IV. Open Problems

Mechanism design in social networks is a very promising research direction, which has not been studied before in the literature of game theory. It also has a broader class of applications around the digital economy and the sharing economy. There are many open problems worth further investigations:

- Diffusion mechanisms for combinatorial settings: we have only looked at simple valuation settings. Whether our methods can be easily extended to more complex settings is an open question. As we have seen from [2], it is already very challenging to move from selling single-item setting to selling multiple-item setting.
- When diffusion is costly: we have assumed that information propagation is not costly, but in real-world applications, users might hesitate to do so, as propagating sale information to their friends might ruin their friendship. If diffusion is costly, can we still guarantee that the seller's revenue is non-decreasing with a diffusion mechanism? We have also considered transfer cost of the items in the network, which is not diffusion cost [5].
- In our setting, we also assumed that the seller is the market owner and she has the whole network structure (after the propagation). Since the seller is aware of the whole network, she can ignore paying other buyers and directly does transactions with the highest buyers. Moreover, buyers may not be confident to reveal their friendship to the seller, which is an important privacy concern in practice.
- Last but not least, buyers can create dummy friends to increase their payments, which is already a very hard problem in classical mechanism design settings [14]. Solving the challenge in our settings seems even harder.

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