

Mobile Location Estimation using a 3-Dimension Ellipse Propagation Model

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Abstract

This is a study of mobile location estimation in three dimension space. We build up a 3-D signal propagation model, namely the three dimension Ellipse Propagation Model (3DEPM), which is an extension of the Ellipse Propagation Model(EPM) from our previous study [1]. Then we use a three dimension Geometric Algorithm (3-D Geometric Algorithm) to calculate the location of the Mobile Station(MS). The 3-D Geometric Algorithm is an extension of the Geometric Algorithm, which can provide a 3-D estimation instead of an estimation on a 2-D Plane. Thus, this study is a meaningful work for locating a mobile station based on the mobile and wireless networks such that it can be a guideline for our future research on mobile computing.

1 Introduction

Many location estimation algorithms based on the radio cellular network have been proposed. These approaches include the Time-Of-Arrival (TOA), Time Difference Of Arrival (TDOA), Enhanced Observed Time Difference (E-OTD) and Angle-Of-Arrival (AOA) [8]. These technologies are based on timing information and angular information. Time based methods, for example, TOA, TDOA and E-OTD, calculate the distance between the Mobile Station (MS) and the Base Station (BS) by measuring the propagation time of the signal and multiply it by the speed of light.

By using trilateration, the position of the MS can be estimated. On the other hand, the AOA approach measures the angle between the MS and the BS and then estimates the MS location by using triangulation. Although these positioning technologies are simple, these approaches are designed for CDMA system only since the system can provide the timing or angular information. Applying these technologies to other cellular network like GSM may require additional hardware and hence increase the implementation cost.

Most of the existing mobile phone operators in Hong Kong have adopted the GSM network instead of the CDMA network. Unlike the CDMA network, the GSM network can only provide the loss of signal strength during signal transmission for positioning purpose [7]. However, the loss of signal strength is the common attributes of all radio cellular network, thus location estimation algorithms based on the signal strength should be applicable to all radio cellular network.

Our group has proposed several location estimation approaches that are based on the received signal strength (RSS) [2, 3, 4]. However, these methods have not included the directional properties. From our observations, we have found that BS has a directional property. That is, BSs always transmit signal in a direction.

In this study, we present a 3-D directional propagation model — the Three Dimension Ellipse Propagation Model (3DEPM). The 3DEPM was derived from the Ellipse Propagation Model (EPM) [1], which is an extension of the EPM. We observed that antenna transmits signal in a direction. Therefore, the contour surface of signal strength should not be modelled as a sphere. Thus, we have modified the original propagation model [8] by considering the contour surface as an ellipse-sphere with the BS at one of the focus. Since the RSS is the only attribute we have which is not the distances between the MS and the BS. Therefore, the 3DEPM does focus on the relationship between the distance and the RSS. Besides the 3DEPM, we have designed a method called the three dimension Geometric Algorithm which is an approach for providing a 3-D location of mobile devices with the 3-D propagation model.

This paper is divided into four sections. In the following section, we will discuss the 3DEPM. In section 3, the three dimension Geometric Algorithm will be presented in details. In section 4, we present the summary and our future work for our research in location estimation.

2 Three Dimension Space Location Model

Although most location estimation algorithms assume a 2-D model, the real location problem is in fact a three dimension space location problem. For example, if we are in a building or in the hilly terrains, we also should consider the attitude besides the 2-D space location. Since our previous work just assume a 2-D model [1], we should extend it into the 3-D space to find out a 3-D signal propagation model for location estimation.

To recall our problem in a 2-D plane, given that we have the locations of three base stations (BSs) and the mobile station (MS) location and its received signal strengths (RSS) from these base stations, we have some methods to find out the relationship of the RSS and the distance between the BS and the MS, then we can provide a solution as the estimation of the MS location.

Suppose the location of four BS locations are known, denoted by $A(\alpha_1, \beta_1, \gamma_1)$, $B(\alpha_2, \beta_2, \gamma_2)$, $C(\alpha_3, \beta_3, \gamma_3)$ and $D(\alpha_4, \beta_4, \gamma_4)$. The RSS from them are denoted by s_1, s_2, s_3 and s_4 with the transmission powers of these BSs are s_1^0, s_2^0, s_3^0 and s_4^0 , respectively. Then the attenuation powers of transmission are $s_1^0 - s_1, s_2^0 - s_2, s_3^0 - s_3, s_4^0 - s_4$.

Let the MS location is $M(x, y, z)$, which is unknown now and it will be estimated by the above information. Given the distances between the BSs and the MS are d_1, d_2, d_3 and d_4 , and we know nothing about these exact values, we just have the RSS and the transmission powers of these BS. Instinctively, it is necessary to find out a model to describe the relationship between the distance and the attenuation power in the transmission. There is a free space signal propagation model to describe this relationship, but in many cases, such as the interference factors are too large and the transmission has a directional property, so this free space propagation model does not model the real situation well. We should submit other models to accommodate these changes of conditions.

Basically, there are many rules in the space transmission, and the free space propagation model is useful but it should be modified. We assume the contour surface is an ellipse-sphere, just like in the free space signal propagation model where the contour surface is a normal-sphere, and we name this model as the 3-Dimension Ellipse Propagation Model.

The ellipse-sphere can be written as a formula for the rectangular coordinates system,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (2.1)$$

where a, b, c are the spindles of the ellipse-sphere, and a is the long spindle.

We rewrite the ellipse-sphere for the three dimension polar coordinates, which can be determined by three factors: distance, horizontal angle and vertical angle, denoted by (r, θ, δ) , and the transform formulas of the polar coordinates and the rectangular coordinates are:

$$\begin{cases} x = r \cos(\theta) \cos(\delta) \\ y = r \sin(\theta) \cos(\delta) \\ z = r \sin(\delta) \end{cases} \quad (2.2)$$

where $\theta \in [0, 2\pi)$ and $\delta \in [-\pi/2, \pi/2)$.

Suppose the origin of the polar coordinates is at one of the focus of the horizontal ellipse and the center of the vertical ellipse, rewriting the formula in the rectangular coordinates as the polar coordinates, then we have a model:

$$r = a * \frac{e_1 \cos(\theta) \cos(\delta) + \sqrt{1 - \frac{e_1^2(1-e_2^2) - e_2^2(1-e_1^2)}{1-e_2^2} \sin^2(\delta)}}{1 + \frac{e_1^2}{1-e_1^2} \sin^2(\theta) \cos^2(\delta) + \frac{e_2^2}{1-e_2^2} \sin^2(\delta)} \quad (2.3)$$

where

$\theta \in [0, 2\pi)$ and $\delta \in [-\frac{\pi}{2}, \frac{\pi}{2})$, which are the horizontal and vertical angels;

e_1 is the eccentricity of the horizontal ellipse;

e_2 is the eccentricity of the vertical ellipse ;

a is the long spindle of the ellipse-sphere;

r is the distance between the MS and BS.

Given that in the main transmission direction, the rule of free space signal propagation works. Namely, in the horizontal level, the rule of free space model does fit. That is:

$$a(1 - e_1) = k(s^0/s)^{1/\alpha}, \quad (2.4)$$

where

k is proportion constant;

s^0 is the transmitting power of the BS;

s is the signal power received from the BS, unit of s^0 and s is watt;

α is a constant of the propagation model and be called the path loss exponent;

and a is the long spindle of the ellipse-sphere.

So Eq.(2.3) can be rewritten as

$$r = k(s^0/s)^{1/\alpha} \frac{\frac{e_1}{1+e_1} \cos(\theta) \cos(\delta) + \frac{1}{1+e_1} \sqrt{1 - \frac{e_1^2(1-e_2^2) - e_2^2(1-e_1^2)}{1-e_2^2} \sin^2(\delta)}}{1 + \frac{e_1^2}{1-e_1^2} \sin^2(\theta) \cos^2(\delta) + \frac{e_2^2}{1-e_2^2} \sin^2(\delta)} \quad (2.5)$$

where

k is proportion constant;

s^0 is the transmitting power of the BS;

s is the received signal power from the BS;

α is the path loss exponent;

$\theta \in [0, 2\pi)$ is the horizontal angle;

$\delta \in [-\pi/2, \pi/2)$ is the vertical angle;

e_1 is the eccentricity of the horizontal ellipse;

e_2 is the eccentricity of the vertical ellipse;

r is the distance between the MS and BS.

We call this relationship as the 3-D Ellipse Propagation Model (3DEPM), in which the contour surface of the signal strength is an ellipse-sphere.

If $\delta = 0$, then Eq.(2.5) can be rewritten as:

$$r = k(s^0/s)^{1/\alpha}(1 - e_1)/(1 - e_1\cos(\theta)). \quad (2.6)$$

This becomes the 2-D Ellipse Propagation Model [1]. So the 3-D model is an extension of the 2-D Ellipse Propagation Model. And if we can use this model to find out the relationship between the distance between the BS and MS and the RSS from the BS. Then we can find out the MS location by a geometric method.

3 The Three Dimension Geometric Algorithm based on the 3DEPM

The 3DEPM is used to translate the RSS into MS-BS distance, while the 3-D Geometric Algorithm is used to estimate the MS location based on the 3DEPM. And the 3-D Geometric Algorithm derives from the Geometric Algorithm [1], which is an algorithm only can find out a 2-D estimation based on the EPM. The 3-D Geometric Algorithm follows the idea of the Geometric Algorithm and provide a 3-D estimation of the MS location.

3.1 Structure of the 3-D Geometric Algorithm

Suppose a MS with location $M(x, y, z)$ received RSS, s_1, s_2, s_3, s_4 from four BSs with locations, $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), (x_4, y_4, z_4)$ respectively. In addition, the distances between MS and the BSs are denoted by $d(s_1), d(s_2), d(s_3)$ and $d(s_4)$, sometimes, they are simply denoted by d_1, d_2, d_3, d_4 . Thus, by the formula of the two points distance in the 3-D Euclidian space, we can form four spheres formulas which are shown as follows,

$$\begin{cases} d_1^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 \\ d_2^2 = (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 \\ d_3^2 = (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 \\ d_4^2 = (x - x_4)^2 + (y - y_4)^2 + (z - z_4)^2 \end{cases} \quad (3.1)$$

Basically, the geometric interpretation of this equation group means four spheres in a 3-D space, and the solution is the intersection point of these spheres. However, the question is if an intersection point does not exist, can we find an estimation for the MS location. In order to give a location estimation every time, the 3-D Geometric Algorithm will not solve the equation group directly. Instead, the 3-D Geometric Algorithm derives another 20 equation groups based

on Equation group (3.1) if these four BSs are not in the same plane. Hence, each new equation group can provide a solution.

For the convenience of our discussion, we just discuss one of the equation groups in detail, which is derived from the Equation group (3.1) with the four BSs that are not in the same plane. Without loss of generally, the other equation groups can be solved in a similar manner. We derive a new equation group by subtracting one equation with another equation in Equation group (3.1).

$$\begin{cases} 2(x_2 - x_1)x + 2(y_2 - y_1)y + 2(z_2 - z_1)z = (x_2^2 + y_2^2 + z_2^2) - (x_1^2 + y_1^2 + z_1^2) + (d_1^2 - d_2^2) \\ 2(x_3 - x_1)x + 2(y_3 - y_1)y + 2(z_3 - z_1)z = (x_3^2 + y_3^2 + z_3^2) - (x_1^2 + y_1^2 + z_1^2) + (d_1^2 - d_3^2) \\ 2(x_4 - x_1)x + 2(y_4 - y_1)y + 2(z_4 - z_1)z = (x_4^2 + y_4^2 + z_4^2) - (x_1^2 + y_1^2 + z_1^2) + (d_1^2 - d_4^2) \end{cases} \quad (3.2)$$

This new equation group can always provide a solution if these four BSs are not in the same plane. By setting,

$$A = \begin{pmatrix} 2(x_2 - x_1), & 2(y_2 - y_1), & 2(z_2 - z_1) \\ 2(x_3 - x_1), & 2(y_3 - y_1), & 2(z_3 - z_1) \\ 2(x_4 - x_1), & 2(y_4 - y_1), & 2(z_4 - z_1) \end{pmatrix} \quad (3.3)$$

and

$$b = \begin{pmatrix} (d_1^2 - d_2^2) - (x_1^2 + y_1^2 + z_1^2) + (x_2^2 + y_2^2 + z_2^2) \\ (d_1^2 - d_3^2) - (x_1^2 + y_1^2 + z_1^2) + (x_3^2 + y_3^2 + z_3^2) \\ (d_1^2 - d_4^2) - (x_1^2 + y_1^2 + z_1^2) + (x_4^2 + y_4^2 + z_4^2) \end{pmatrix} \quad (3.4)$$

and

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (3.5)$$

Equation group (3.2) can be rewritten into the matrix form,

$$AX = b \quad (3.6)$$

If $|A| \neq 0$, the only solution of the matrix equation is,

$$X = A^{-1}b \quad (3.7)$$

If $|A|$ does not equal to zero, then the Equation group (3.6) has a real root. However, when the four BSs lie on the same plane in the 3-D space, and that the four BSs lie on one straight line

is included in this case, $|A|$ will be equal to zero. Hence, we can move the location of these BSs a little bit in order to satisfy $|A| \neq 0$, then we can use Eq. (3.6) to find out the estimation.

As we have these $(C_6^3 - 1)$ equation groups similar to the Equation group (3.2), thus, we will have 20 solutions. By calculating the centroid of these solutions, we can have an estimation of the MS location.

3.2 The Explanation of the 3-D Geometric Algorithm

The 3-D Geometric Algorithm uses four distances between the MS location and the locations of BSs to provide the MS location in the 3-D space. Usually, in 3-D space, it is necessary to use three distances to estimate the MS location. That is, it is necessary to use three spheres to find out the MS location. But in this case, there does not always exist a certain solution, since these three sphere may not always have only one intersection.

In order to solve this ambiguous problem, the 3-D Geometric Algorithm used three spheres formed by these four receiving BSs to estimate the MS location. In this case, we will always obtain a solution if the four receiving BSs do not lie on the same plane. Since we can form a plane equation by subtracting one sphere with another sphere. Therefore, we can obtain C_6^3 combinations of planes. And each combination can only derive one intersection point if and only if these four receiving BSs are not in the same plane, that is, these three planes in one combination are not parallel. Thus, we will have 20 intersections and we can estimate the position of the MS as the centroid of these intersections.

3.3 Steps of the 3-D Geometric Algorithm

The steps of the 3-D Geometric Algorithm can be described as follows:

- Step 1: Given values of parameters of 3DEPM, the RSS, s , and the transmission angle of the BS, θ, δ ;
- Step 2: Compute the distances between the MS and BSs by the 3DEPM;
- Step 3: Compute the estimation of the MS location (x, y, z) by the 3-D Geometric Algorithm;
- Step 4: Output (x, y, z) as the estimation of MS location.

4 Summary and Future work

We have presented a three dimension signal propagation model—the three dimension Ellipse Propagation Model(3DEPM) and a method to provide a 3-D estimation based on this 3-D model,

namely by the three dimension Geometric Algorithm. This 3-D model and the method to provide the MS location are the extension of the Ellipse Propagation Model(EPM) [1]. Since the EPM is just for the 2-D model and the estimation derived from it is also a 2-D solution. They do not meet the real situation in the real 3-D world. In order to provide a better location service, a 3-D estimation must be found. This is our motivation to build up a 3-D signal propagation model for location estimation.

The three dimension signal propagation model has better accuracy than the EPM, since the EPM just used the BS location in a plane to estimate the MS location, and it considered nothing about the height of the antenna, while the 3-D model uses the three dimension location of the BS. And by our research, the height of the antenna is also an important factor for locating to the MS, since an antenna in the peak has different effect with the one in Central for a MS. And the special hilly terrains in Hong Kong also demand a 3-D location service. So the three dimension signal propagation model and the three dimension estimation derived from it can provide a better location service for the Hong Kong mobile user.

Since our previous before research had not focused on the 3-D MS location, and the field test data were just 2-D data, this research study can be a guideline for our future study. We can design our field test plan based on the 3-D model and take the 3-D data, then build up a 3-D signal propagation model to provide a more accurate estimation of MS location.

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