An Iterative Approach to Mobile Location Estimation

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keywords Iterative approach, Mobile location estimation, Base station, Signal strength

Abstract

Mobile Location Estimation is drawing considerable attention in the field of wireless communications. In this research study, we present an iterative algorithm to get a stable estimation based on the Ellipse Propagation Model (EPM). The iterative algorithm based on the geometric algorithm derives from statistical analysis. It can modify the defect of the geometric algorithm approach and get a more stable and accurate estimation on the location of a mobile station(MS).

1 Introduction

Mobile Location Estimation is receiving considerable attention in the field of wireless communications. How to estimate the location of a mobile station becomes an interesting problem. There are many algorithms and models proposed for mobile location estimation. J.Y.Zhou has presented a directional model—the Ellipse Propagation Model, and one geometric algorithm based on the EPM[1]. Joseph Ng, Kenny K.H. Kan , Stephan K.C. Chan and S.B. Song have presented different algorithms which base on the signal attenuation rule [2, 3, 4]. Kenneth M. Chu and Teemu Roos also focus on the model based on the directional transmission and use the view of probability and statistical to build up the propagation model and provide the estimation [5, 6]. But all the algorithms mentioned above get an unstable estimation, which have no self-modification property.

In this study, we present an iterative algorithm based on the EPM [1]. It is an algorithm based on the signal attenuation rule. The idea of iterative algorithm derives from **"The Browser Theorem"**. At first, we use one of the solutions derived by [1], [2], [3] or [4] as an initial value for our iterative approcach, then we use the iterative formula to get a new solution and choose the convergence point as the estimation. We can proof that the iterative formula has convergence under some conditions, in fact, these conditions can easily be satisfied. The iterative algorithm is useful to provide location estimation, which is more stable and more accurate than our previous methods.

2 The Iterative Algorithm based on the Ellipse Propagation Model

There are many methods proposed to solve the MS positioning problem, such as [1, 2, 3, 4, 5, 6]. However, these methods have no self-modification property. One intuitive thread is to use the iterative method, then choose the convergence value for location estimation. So we designed an Iterative Algorithm for the location estimation of MS, and its structure is derived from the Geometric Algorithm.

2.1 Structure of the Iterative Algorithm

Suppose the coordinates of the MS location, x and y are two independent random variables. We consider the estimation of the MS location as a conditional expectation of the RSS and the BSs locations, denoted by,

$$\begin{cases} x' = E(x|x_0, y_0; s; l) \\ y' = E(y|x_0, y_0; s; l) \end{cases} \tag{2.1}$$

where

x_0 and y_0 are the location of MS we want to find out;
s is the information of RSS;
l is the information of BSs;
x' and y' are random variables.

By rewriting x and y as two parts, we have,

$$\begin{cases} x = f(x_0, y_0; s; l) + \epsilon \\ y = g(x_0, y_0; s; l) + \eta \end{cases} \tag{2.2}$$

where

$f(x_0, y_0; s; l)$ and $g(x_0, y_0; s; l)$ are the certain terms;
x_0 and y_0 are the estimations of x and y;

ϵ and η are random variables;

f and g are some functions structures, which have first order and second order derivatives.

Furthermore, we assume that $E(\epsilon) = 0$, $E(\eta) = 0$, $var(\epsilon) = \sigma_\epsilon^2$, $var(\eta) = \sigma_\eta^2$, $cov(\epsilon, \eta) = 0$, x_0 and y_0 are unbiased estimations. Thus, $E(x - x_0) = 0$, $E(y - y_0) = 0$.

In order to choose the best estimation, a criteria is needed for the comparison between x, y and x_0, y_0.

We define

$$d(x, y; x_0, y_0) \stackrel{def}{=} (x - f(x_0, y_0; s; l))^2 + (y - g(x_0, y_0; s; l))^2 \tag{2.3}$$

Our task is to find out an estimation of $(\hat{x}, \hat{y})$ which satisfies,

$$E(d(\hat{x}, \hat{y}; x_0, y_0)) = min(E(d(x, y; x_0, y_0))) \tag{2.4}$$

Since x and y are random variables, so $d(x, y; x_0, y_0)$ is also a random variable. By comparing their expectations, we have,

$$\begin{aligned} d(x, y; x_0, y_0) &= (x - f(x_0, y_0; s; l))^2 + (y - g(x_0, y_0; s; l))^2 \\ &= ((x - x_0) + (x_0 - f(x_0, y_0; s; l)))^2 \\ &\quad +((y - y_0) + (y_0 - g(x_0, y_0; s; l)))^2 \end{aligned}$$

Set $E = E(d(x, y; x_0, y_0))$, thus,

$$E = \sigma_\epsilon^2 + (x_0 - f(x_0, y_0; s; l))^2 + \sigma_\eta^2 + (y_0 - g(x_0, y_0; s; l))^2 \tag{2.5}$$

$x_0 = f(x_0, y_0; s; l)$ and $y_0 = g(x_0, y_0; s; l)$ satisfy,

- $\frac{\partial E}{\partial x_0} = 0$ and $\frac{\partial E}{\partial y_0} = 0$
- $\frac{\partial^2 E}{\partial x_0^2} > 0$ and $\frac{\partial^2 E}{\partial y_0^2} > 0$
- $\frac{\partial^2 E}{\partial x_0 \partial y_0} = \frac{\partial^2 E}{\partial y_0 \partial x_0} = 0$

then $d(x, y; x_0, y_0)$ reaches the local minimum. And they can be rewritten as,

$$\begin{cases} x_0 = f(x_0, y_0; s_1, s_2, ..., s_m; l_1, l_2, ..., l_m) \\ y_0 = g(x_0, y_0; s_1, s_2, ..., s_m; l_1, l_2, ..., l_m) \end{cases} \tag{2.6}$$

Since x_0 and y_0 appear in both sides of the two equations, one can use the iterative method to find out the solution.

Therefore, by using an initial value, for example, the estimation from the CG algorithm (x_{CG}, y_{CG}), we can get the convergence value as the estimation of MS if the iterative formulas are convergent.

2.2 Using Geometric Algorithm with the Iterative Algorithm

¿From the simple signal propagation model [8], the relationship between the RSS, s and the distance, d is,

$$s = s_0(d/d_0)^{-\alpha} = kd^{-\alpha} \tag{2.7}$$

where

s_0 is the power received at a reference distance;
d_0 is the reference distance;
α is called the path loss exponent;
and k is a constant.

Suppose a MS received RSS, s_1, s_2, s_3 from three BSs with locations, (α_1, β_1), (α_2, β_2), (α_3, β_3) with output power, o_1, o_2, o_3 respectively. In addition, the distance between the MS and the BSs are denoted by $d(s_1)$, $d(s_2)$ and $d(s_3)$, sometimes they are simply denoted by d_1, d_2 and d_3. From the Iterative Algorithm, we have,

$$\begin{cases} x = f(s_1, s_2, s_3; l_1, l_2, l_3) \\ y = g(s_1, s_2, s_3; l_1, l_2, l_3) \end{cases} \tag{2.8}$$

Also, using the Geometric Algorithm and the EPM [1],

$$\begin{cases} x = (2m(\beta_3 - \beta_1) - 2n(\beta_2 - \beta_1))/|A| \\ y = (-2m(\alpha_3 - \alpha_1) + 2n(\alpha_2 - \alpha_1))/|A| \end{cases} \tag{2.9}$$

where

$|A| = 4[(\alpha_2 - \alpha_1)(\beta_3 - \beta_1) - (\alpha_3 - \alpha_1)(\beta_2 - \beta_1)]$;
$m = (\alpha_2^2 + \beta_2^2) - (\alpha_1^2 + \beta_1^2) + (d_1^2 - d_2^2)$;
$n = (\alpha_3^2 + \beta_3^2) - (\alpha_1^2 + \beta_1^2) + (d_1^2 - d_3^2)$;
$d_1 = (o_1/s_1)^{1/\alpha}(1 - e_1)/(1 - e_1 \cos(\theta_1))$;
$d_2 = (o_2/s_2)^{1/\alpha}(1 - e_2)/(1 - e_2 \cos(\theta_2))$;
$d_3 = (o_3/s_3)^{1/\alpha}(1 - e_3)/(1 - e_3 \cos(\theta_3))$;

We Define θ as

$$\theta = \begin{cases} \frac{5\pi}{2} - bear - \arccos(\frac{x-\alpha}{\sqrt{(x-\alpha)^2+(y-\beta)^2}}) & \text{if } y > \beta \\ \frac{\pi}{2} - bear + \arccos(\frac{x-\alpha}{\sqrt{(x-\alpha)^2+(y-\beta)^2}}) & \text{if } y \leq \beta \end{cases} \tag{2.10}$$

where

θ is the deviation,which contains the bearing information;

$bear$ is the bearing information;

s is the received signal power;

o is the transmitter power of the BS;

e is the eccentricity of the ellipse;

α is the path loss exponent.

So the iterative formulae become,

$$\begin{aligned} x_{n+1} &= 2[(\beta_3 - \beta_2)(d_1^2(n) - (\alpha_1^2 + \beta_1^2)) \\ &\quad +(\beta_1 - \beta_3)((d_2^2(n)) - (\alpha_2^2 + \beta_2^2)) \\ &\quad +\beta_2 - \beta_1)(d_3^2(n) - (\alpha_3^2 + \beta_3^2))]/|A| \\ y_{n+1} &= 2[(\alpha_2 - \alpha_3)(d_1^2(n) - (\alpha_1^2 + \beta_1^2)) \\ &\quad +(\alpha_3 - \alpha_1)((d_2^2(n)) - (\alpha_2^2 + \beta_2^2)) \\ &\quad +(\alpha_1 - \alpha_2)(d_3^2(n) - (\alpha_3^2 + \beta_3^2))]/|A| \end{aligned} \tag{2.11}$$

Thus,

$$\begin{aligned} x_{n+1} &= f(x_n, y_n; s_1, s_2, s_3; l_1, l_2, l_3), \\ y_{n+1} &= g(x_n, y_n; s_1, s_2, s_3; l_1, l_2, l_3). \end{aligned} \tag{2.12}$$

If (x_n, y_n) converges to one point $(\hat{x}, \hat{y})$, then $(\hat{x}, \hat{y})$ is considered to be the location estimation of the MS.

2.3 Convergence of the Iterative algorithm

The Iterative Algorithm is convergence under some conditions, and we present these conditions as a theory. Define,

$$d(x, y, \alpha_l, \beta_l, \alpha, s, e) = (s_0/s)^{1/\alpha}(1-e)/(1-e\cos(\theta)) \tag{2.13}$$

$$de(x, y, \alpha_l, \beta_l) = \sqrt{(x-\alpha_l)^2 + (y-\beta_l)^2} \tag{2.14}$$

where, $d(x, y, \alpha_l, \beta_l, \alpha, s, e)$ is the distance function of the EPM and $de(x, y, \alpha_l, \beta_l)$ is the common distance function in Euclidean space. To simplify our discussion, if we fix one parameter, for example, we fix y for x, then we denote them as $d(x)$ and $de(x)$. We assume the estimated distance between a MS and the i^{th} BS using EPM is $d_i(x, y)$. We can form three circles for three BS,

$$(x - x_1)^2 + (y - y_1)^2 = d_1^2(x, y) \tag{2.15}$$

$$(x - x_2)^2 + (y - y_2)^2 = d_2^2(x, y) \tag{2.16}$$

$$(x - x_3)^2 + (y - y_3)^2 = d_3^2(x, y) \tag{2.17}$$

If three circles intersected at one point, then $d(x,y) = de(x,y)$. Otherwise, we suppose that $d(x,y) \leq de(x,y)$.

Theorem 2.1 *If* (α_1, β_1), (α_2, β_2), (α_3, β_3) *are not in the same line, and suppose that*

$$|\frac{d_i^2(x,y)}{de_i^2(x,y)} \cdot \frac{(y-\beta_i)}{de_i(x,y)} \cdot \frac{e_i \sin(\theta_i)}{1 - e_i \cos(\theta_i)}| \leq 1,$$

and

$$|\alpha_i - \alpha_i| \geq 4, if|\alpha_i - \alpha_i| \neq 0;$$

$$|\beta_i - \beta_i| \geq 4, if|\beta_i - \beta_i| \neq 0;$$

where $i \neq j$ *and* $i, j = 1, 2, 3$.

Then the iterative formula $\{x_n\}$ *and* $\{y_n\}$ *have convergence.*

Proof: Rewrite the equation as the matrix formula,

$$AX = b \tag{2.18}$$

where

$$X = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.19}$$

Suppose $\alpha_1 \leq \alpha_3 \leq \alpha_2$ and $\beta_2 \leq \beta_1 \leq \beta_3$. If $\alpha_1 = \alpha_3$, we add the condition, $\frac{\beta_1 - \beta_2}{\beta_3 - \beta_2} \leq \frac{1}{2}$. Set

$$A = \begin{pmatrix} 2(\alpha_2 - \alpha_1), & 2(\beta_2 - \beta_1) \\ 2(\alpha_3 - \alpha_1), & 2(\beta_3 - \beta_1) \end{pmatrix} \tag{2.20}$$

and

$$b = \begin{pmatrix} d_1^2 - d_2^2 - (\alpha_1^2 + \beta_1^2) + (\alpha_2^2 + \beta_2^2) \\ d_1^2 - d_3^2 - (\alpha_1^2 + \beta_1^2) + (\alpha_3^2 + \beta_3^2) \end{pmatrix} \tag{2.21}$$

Since (α_1, β_1),(α_2, β_2) ,(α_3, β_3) are not in the same line, so $|A| \neq 0$. And the solution is

$$X = A^{-1}b$$

Rewrite it as the iterative formula.

$$X_{n+1} = A^{-1}b(X_n)$$

That is

$$
\begin{aligned}
x_{n+1} = 2[&(\beta_3-\beta_2)(d_1^2(n)-(\alpha_1^2+\beta_1^2))\\
&+(\beta_1-\beta_3)((d_2^2(n))-(\alpha_2^2+\beta_2^2))\\
&+(\beta_2-\beta_1)(d_3^2(n)-(\alpha_3^2+\beta_3^2))]/|A|\\
y_{n+1} = 2[&(\alpha_2-\alpha_3)(d_1^2(n)-(\alpha_1^2+\beta_1^2))\\
&+(\alpha_3-\alpha_1)((d_2^2(n))-(\alpha_2^2+\beta_2^2))\\
&+(\alpha_1-\alpha_2)(d_3^2(n)-(\alpha_3^2+\beta_3^2))]/|A|
\end{aligned}
$$

Since,

$$d_1(n) = (o_1/s_1)^{1/\alpha}(1-e_1)/(1-e_1\cos(\theta_1^{(n)})) \tag{2.22}$$

$$d_2(n) = (o_2/s_2)^{1/\alpha}(1-e_2)/(1-e_2\cos(\theta_2^{(n)})) \tag{2.23}$$

$$d_3(n) = (o_3/s_3)^{1/\alpha}(1-e_3)/(1-e_3\cos(\theta_3^{(n)})) \tag{2.24}$$

Thus,

$$(o_1/s_1)^{1/\alpha}(1-e_1)/(1+e_1) \le d_1(n) \le (o_1/s_1)^{1/\alpha} \tag{2.25}$$

$$(o_2/s_2)^{1/\alpha}(1-e_2)/(1+e_2) \le d_2(n) \le (o_2/s_2)^{1/\alpha} \tag{2.26}$$

$$(o_3/s_3)^{1/\alpha}(1-e_3)/(1+e_3) \le d_3(n) \le (o_3/s_3)^{1/\alpha} \tag{2.27}$$

So both $\{x_n\}$ and $\{y_n\}$ have bounds.

We fix y when x_n changes, and fix x when y_n changes. In this study, the convergence of $\{x_n\}$ will be discussed in detail while $\{y_n\}$ is just a similar case.

Set

$$f(x) = (\beta_3-\beta_2)d_1^2(x)+(\beta_1-\beta_3)d_2^2(x)+(\beta_2-\beta_1)d_3^2(x)$$

and

$$(x_{n+1}-x_n)|A|/2 = f(x_n)-f(x_{n-1}).$$

By **the Lagrange Theory** [7], we have

$$f(x_n)-f(x_{n-1}) = f^{'}(x)(x_n-x_{n-1}) \tag{2.28}$$

where x is between x_n and x_{n-1}.

And

$$
\begin{aligned}
\frac{f^{'}(x)}{2} = & [\tfrac{(\beta_3-\beta_2)\cdot d_1^2(x)\cdot(y-\beta_1)\cdot e_1\sin(\theta_1)}{de_1^3(x)\cdot(1-e_1\cos(\theta_1))}\\
&+\tfrac{(\beta_1-\beta_3)\cdot d_2^2(x)\cdot(y-\beta_2)\cdot e_2\sin(\theta_2)}{de_2^3(x)\cdot(1-e_2\cos(\theta_2))}\\
&+\tfrac{(\beta_2-\beta_1)\cdot d_3^2(x)\cdot(y-\beta_3)\cdot e_3\sin(\theta_3)}{de_3^3(x)\cdot(1-e_3\cos(\theta_3))}]
\end{aligned} \tag{2.29}
$$

Set $|2f^{'}(x)/|A|| = K$, namely, $K \geq 0$ and $|A| = (\alpha_2 - \alpha_1)(\beta_3 - \beta_1) - (\alpha_3 - \alpha_1)(\beta_2 - \beta_1)$.

If $\alpha_3 - \alpha_1 \neq 0$ then $\alpha_3 - \alpha_1 \geq 4$.

By the conditions,

$$K < \frac{2}{(\alpha_2 - \alpha_1)} + \frac{2}{\alpha_3 - \alpha_1} \leq \frac{4}{\alpha_3 - \alpha_1} \leq 1.$$

If $\alpha_3 - \alpha_1 = 0$,

$$K < \frac{2}{(\alpha_2 - \alpha_1) - (\alpha_2 - \alpha_1)\frac{\beta_1 - \beta_2}{\beta_3 - \beta_2}} \leq \frac{4}{\alpha_2 - \alpha_1} \leq 1$$

That is , $K < 1$. And $(x_{n+1} - x_n) = K(x_n - x_{n-1})$.

So $\{x_n\}$ has convergence. Similarly, $\{y_n\}$ also has convergence.

Q.E.D.

2.4 Steps of The Iterative Algorithm

The Iterative Steps can be described as follows:

Step 1: Given an estimation value (x_n, y_n) (if $n = 0$, it is the initial value), the signal strength s and the model parameter value;

Step 2: Fixed y for x, compute the angle, $\theta_{i,1}^{(n)}$, between the estimation location and the i^{th} base station;

Step 3: Compute $d_i(\theta_{i,1}^{(n)})$ by the EPM;

Step 4: Compute x_{n+1};

Step5 : Fixed x for y, compute the angle,$\theta_{i,2}^{(n)}$, between the estimation location and the base station again;

Step 6: Compute $d_1(\theta_{1,2}^{(n)})$ by the EPM;

Step 7: Compute y_{n+1};

Step 8: If the error between (x_{n+1}, y_{n+1}) and (x_n, y_n) meets some conditions, use (x_{n+1}, y_{n+1}) as the estimation of the location of mobile station. Otherwise, set $(x_{n+1}y_{n+1})$ as (x_n, y_n) and go back to step 2.

3 Simulation Results

We have conducted field test in many regions in Hong Kong in order to validate our model. The data we collected from the field test is first used to find out the EPM parameters. These parameters are then put into a *Lookup Table* which will be used during the testing process. In the testing phase, we apply the Geometric Algorithm and the Iterative Algorithm with the data we have collected to compute the location of the MS. The results are then compare with the CG and CT algorithms.

3.1 Estimating the EPM parameters

We choose the whole field test data to find out the EPM parameters. For saving the computation cost, we categorize all types of BSs into three groups in each region, we denote them by Macro, Micro and others. Since the environment condition is similar within a region, we assume the value of the path loss exponent, α, is the same throughout the region. Thus, there will be four parameters needed to be trained in each region, e_1, e_2, e_3 and α, where e_1, e_2 and e_3 are the EPM parameters of Macro, Micro and others respectively for a region. We set the incremental step for the eccentricity, e_1, e_2 and e_3 to 0.1, and the incremental step for the path loss exponent, α, to 0.1 for saving computational costs. As α is expected to be within a range as suggested by [8], therefore, we choose α to vary within a range of 3 and 10 in order to cover the situation in Hong Kong. On the other hand, the eccentricity of a ellipse has its natural limitation, it can only vary within a range of 0 and 1. We find out the values of these four parameters for each region and record them into a *Lookup Table* as shown in Table 1 below. The *Lookup Table* contains information of the EPM in different region and will be used during the estimation process. As we can observe from Table 1, the value of the EPM parameters vary from region to region because each region has its own features, such as high buildings, hilly terrains, sea shores and so on.

Region	e_1	e_2	e_3	α	Region	e_1	e_2	e_3	α
Aberdeen	0.3	0	0	6.4	ShamTseng	0.9	0.1	0	5.3
CauseWayBay	0.5	0.9	0	4.2	ShaTin	0.5	0.8	0	5.9
Central	0	0.9	0	6.5	ShekKipMeiPark	0.8	0	0	5.8
CheungShaWan	0	0.6	0	8.8	SheungShui	0	0	0	6.9
FoTan	0.9	0	0	6.3	SheungWan	0	0.9	0	7.1
HappyValley	0.7	0	0	6.8	TaiKooShing	0.9	0	0	5.4
HungHom	0.8	0	0	7	TaiWai	0.3	0	0	6.4
KowloonBay	0	0	0.4	8.7	TaiWoHau	0.2	0	0	5.8
KowloonCity	0.6	0.7	0	6.2	TinShuiWai	0.9	0	0	5.6
KowloonTong	0	0	0.4	7.2	ToLoHighway	0.1	0	0	5.4
KwaiFong	0.9	0.9	0	6.1	TsingYi	0.1	0	0	7
KwunTong	0	0	0	8.2	TsuenWan	0.9	0.9	0	8.9
LaiChiKok	0	0	0	7.8	TszWanShan	0.1	0	0	7.1
LaiKing	0.6	0.9	0.5	7.1	TuenMun	0.4	0	0.4	6.4
MaOnShan	0	0	0	8.9	WanChai	0.1	0.9	0.9	7
Mongkok	0.5	0.9	0.1	5.3	WongTaiSin	0.9	0	0	7.2
PE-MK	0.4	0.9	0.1	8.9	YauTong	0.1	0	0	6.2
PrinceEdward	0	0.9	0	7.9	YauYatChuen	0.3	0	0	7
ShamShuiPo	0.4	0.3	0.7	8.9	YuenLong	0.9	0	0	5.3

Table 1: The Lookup Table

3.2 Results of the Iterative Algorithm based on the EPM

After constructing the *Lookup Table*, the method of the Iterative Algorithm is used to estimate the location of the MS. Since each region has its own feature and terrain, we use different criteria to choose the estimation of the MS location in different region in the model computation. We present not only the mean value and its standard deviation, but also the 67% value point and the 90% value point to describe the estimation in greater details.

The iterative algorithm will derive a stable estimation, since it has a convergence point. Since other algorithms, such as those proposed in [1, 2, 3, 4], while Kenneth and Teemu can just provide an unstable estimation[5] and [6], the iterative algorithm can provide an estimation that is closer to the marker location by comparing with the result of [1].

The iterative algorithm is more computational intensive for the mobile location estimation than the geometric algorithm [1]. Although more computational intensive, the iterative algorithm has better estimation error than the other methods. In orde to save the computational cost, we choose the criteria of the convergence is 5 meter, that is to say, if the error of two continues iterative values is within the criteria of the convergence condition, we choose one of the iterative value as the estimation of the MS location. Results are shown in Table 2.

Region	Aver.	Std.	67%	90%	Region	Aver.	Std.	67%	90%
Aberdeen	233.21	140.20	248.73	491.38	ShamTseng	1896.60	855.40	2531.23	3022.95
CauseWayBay	247.97	188.23	307.71	614.57	ShaTin	318.89	160.89	409.60	614.93
Central	106.14	63.97	139.86	195.63	ShekKipMeiPark	310.25	186.34	358.65	574.62
CheungShaWan	120.55	68.59	156.68	212.46	SheungShui	492.11	307.42	803.50	888.64
FoTan	250.58	133.61	299.43	435.65	SheungWan	116.12	63.52	151.56	198.76
HappyValley	332.06	216.64	387.05	605.80	TaiKooShing	189.37	120.11	218.64	374.64
HungHom	930.25	531.46	1366.67	1598.50	TaiWai	225.82	90.62	268.21	319.17
KowloonBay	201.41	107.25	233.14	328.65	TaiWoHau	248.50	134.61	284.51	486.79
KowloonCity	221.56	129.57	259.86	504.56	TinShuiWai	375.94	195.22	490.20	655.78
KowloonTong	252.24	155.48	311.70	498.34	ToLoHighway	915.57	609.12	1352.25	2259.92
KwaiFong	187.06	103.85	233.06	329.67	TsingYi	484.75	240.36	586.58	838.97
KwunTong	113.86	60.43	133.78	217.37	TsuenWan	145.34	60.38	159.47	234.46
LaiChiKok	328.71	197.89	403.47	599.42	TszWanShan	240.37	126.59	288.68	433.32
LaiKing	776.12	308.10	962.79	1287.12	TuenMun	314.75	150.53	403.99	499.47
MaOnShan	356.42	122.84	404.56	486.20	WanChai	167.04	109.52	187.42	320.69
Mongkok	92.49	52.43	116.39	158.34	WongTaiSin	257.55	141.22	314.06	468.18
PE-MK	110.43	75.53	134.88	201.60	YauTong	452.45	320.34	521.41	820.10
PrinceEdward	120.60	73.07	136.07	225.67	YauYatChuen	226.42	108.00	274.67	378.60
ShamShuiPo	104.67	58.75	122.54	195.54	YuenLong	237.57	243.58	214.91	769.39

Table 2: Result for each region with Iterative Algorithm (Unit: meter)

3.3 Compare with the CG ,CT and Geometric algorithms

We compare the results of CG ,CT algorithms and the Geometric Algorithm [1] with the algorithm we present in this research study: the Iterative Algorithm. We give the mean and the standard deviation of the errors to show their estimation effect. And we show the results in Table 3.

Table 3 shows the average and the standard deviation of the error for all the regions. Based on the results in Table 3, we can conclude that the Geometric Algorithm and Iterative Algorithm have better effect than CG [2] and CT [3]. Notice that CT has the smallest standard deviation on the grounds but the missing ration of the CT is high. As a whole, The Geometric algorithm and the Iterative algorithm have great improvement on locating the MS in a radio cellular network.

Model	Average Error	Standard Deviation	sample number	successful %
CG	335.35	319.61	96207	92.6%
CT	321.62	229.06	96207	81.5%
Geometric Algorithm	285.27	309.80	96207	98.3%
Iterative Algorithm	282.68	309.05	96207	98.3%

Table 3: Compare with CG, CT, Geometric and Iterative Algorithm (Unit: meter)

4 Conclusions and Future work

The Iterative Algorithm based on the EPM derives from the Geometric Algorithm [1]. Simulation results show that the Geometric Algorithm and the Iterative Algorithm have better accuracy than the CG and CT algorithms. And the iterative algorithm has better result than the Geometric Algorithm. The iterative algorithm can get a more stable estimation, while the geometric algorithm only can get an unstable estimation. And the overall result of the Iterative Algorithm is better than the Geometric by comparing with some different statistical points.

But the defect of the Iterative Algorithm is the computational cost. Since it follows the estimation by iterative method, most computational cost has been added in the system. And it will cost more time to get the location estimation. By application, we can first supply the estimation of geometric algorithm to the user, and then supply the iterative estimation to reduce the computation time. On another hand, the Iterative Algorithm just has the convergence point under some conditions. That is to say, sometimes we have no convergence point. We have to choose one equilibrium point between the accurate estimation and the computational cost. We think the precision is important to the estimation. And the iterative algorithm can provide a better result than the geometric algorithm based on the EPM.

During this research, we found that signal do fluctuate at the same places. The signal attenuation can be affected by conditions, such as weather and car movement. The fluctuating signal will induce more error in our estimation. As for our future work, we will try to find out a filtering method to handle the problem of signal fluctuation. In addition, we would like to extend the EPM to a 3-dimension model in order to meet the real world situation on mobile computing.

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