Image Analysis Based on the Local Features of 2D Intrinsic Mode Functions

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Abstract

The Hilbert-Huang transform (HHT) is a novel signal processing method which can efficiently handle non-stationary and nonlinear signals. It firstly decomposes signals into a series of Intrinsic Mode Functions (IMFs) adaptively by the Empirical Mode Decomposition (EMD), then applies the Hilbert transform on the IMFs afterward. Based on the analytical signals obtained, the local analysis of the IMFs are conducted. This paper contains two main works. First, we proposed a new two-dimensional EMD (2DEMD) method, which is faster, better-performed than the current 2DEMD methods. Second, the Riesz transform are utilized on the 2DIMFs to get the 2D analytical signals. The local features (amplitude, phase orientation, phase angle, etc) are evaluated. The performances are demonstrated on both texture images and natural images.

1 Introduction

Texture [1] is ubiquitous and provides powerful characteristics for many image analysis applications such as segmentation, edge detection, and image retrieval. Among various texture analysis methods, signal processing methods are promising, which include Gabor filters [2], wavelet transforms [3], Wigner distributions and so forth. They characterize textures through filter responses directly.

Hilbert-Huang transform is a new signal processing method proposed by Huang et al [8, 10, 9]. It contains two kernel parts: Empirical Mode Decomposition (EMD) and Hilbert transform. First, it decomposes signals into a series of Intrinsic Mode Functions (IMFs) adaptively by the Empirical Mode Decomposition (EMD), then applies the Hilbert transform on the IMFs afterward. Based on the analytical signals obtained, the local analysis of the IMFs are conducted. Though Fourier spectral analysis and wavelet transform have provided some general methods for analyzing signals and data, they are still weak at non-stationary and nonlinear data processing. However, due to the fully data-driven process, the HHT is more efficient in this situation. It provides an efficient way for the local analysis.

As the kernel part of HHT, EMD works as a filter bank. It has a wide application in signal analysis including ocean waves, rogue water waves, sound analysis, earthquake time records as well as image analysis [19, 18, 16, 21, 20, 22, 23]. The EMD has been extended to 2DEMD. However, it countered a lot of difficulties such as inaccuracy of surface interpolation, high computational complexity and so forth [27, 29, 25, 26, 28, 31]. Finding a powerful 2DEMD is still a challenge. In this paper, we implemented a modified 2DEMD and study the local properties of the 2DIMFs by Riesz transform [4, 5]. The image after its Riesz transform, we can get the 2D analytical signal. The estimation of the local features is crucial in image processing. Generally, structures such as lines and edges can be distinguised by the local phase, the local amplitude can be used for edge detection.

This paper is organized as follows: Section 2 presents an introduction to HHT. In Section 3, the details of the modified 2DEMD are shown firstly, then we reviewed the Riesz transform. The simulation results are demonstrated in Section 4. Finally, the conclusions are given.

2 Hilbert-Huang transform

Hilbert-Huang transform was proposed by N.E.Huang in 1998. It contains two parts in terms of Empirical Mode Decomposition (EMD) and Hilbert transform. The signals are decomposed into a series of Intrinsic Mode Functions (IMFs), then Hilbert transform are applied on these IMFs to get analytic signals. Since this method is local, data-driven, it is capable of handling nonlinear and non-stationary signals.

EMD captures information about local trends in the signal by measuring oscillations, which can be quantized by a local high frequency or a local low frequency, corresponding to finest detail and coarsest content. Here we briefly review the sifting process of EMD. Four main steps are contained, S1, S2, S3 and S4 are abbreviation for Step 1 to Step 4. Given a signal x(t),

S1. Identify all the local minima and maxima of the input signals $x(t)$;

S2. Interpolate between all minima and maxima to yield two corresponding envelopes $E_{max}(t)$ and $E_{min}(t)$. Calculate the mean envelope $m(t) = (E_{max}(t) + E_{min}(t))/2$;

S3. Compute the residue $h(t) = x(t) - m(t)$. If it is less than the threshold predefined then it becomes the first IMF, go to Step 4. Otherwise, repeat Step 1 and Step 2 using the residue $h(t)$, until the latest residue meets the threshold and turns to be an IMF;

S4. Input the residue r(t) to the loop from Step 1 to Step 3 to get the next remained IMFs until it can not be decomposed further.

The analytical signal provides a way to compute the 1D signal's local amplitude and phase, which is obtained by the Hilbert transform on a real signal. The Hilbert transform $f_H(x)$ of a real 1D signal f is given by:

$$f_H(x) = f(x) * \frac{1}{\pi x},$$

where $*$ is convolution. $f_H(x)$ is the imaginary part of the signal. The analytical signal can be written as

$$f_A = f(x) + i f_H(x) = a(t)e^{i\theta(t)},$$

in which, $a(t)$ is the amplitude, $\theta(t)$ is the phase.

Applying Hilbert transform on each IMF can evaluate the local properties such as amplitude and phase.

3 Local Analysis of 2DIMFs

3.1 The improved 2DEMD

Here we propose an alternative algorithm for EMD. Instead of using the envelopes generated by splines we use a low pass filter to generate a "moving average" to replace the mean of the envelopes. The essence of the sifting algorithm remains.

The moving average is the most common filter in digital signal processing. It operates by averaging a number of points from the input signal to produce each point in the output signal, it is written:

$$y[i] = \frac{1}{M} \sum_{j=0}^{M-1} x[i+j],$$

where $x[]$ is the input signal, $y[]$ is the output signal, and M is the number of points used in the moving average. It is actually a convolution using a simple filter $[a_i]_{i=1}^{M}, a_i = \frac{1}{M}$, and $[A_{i,j}]_{i=1,j=1}^{M,N}, A_{i,j} = \frac{1}{M \times N}$ for the 2-dimensional case.

Detection of local extrema means finding the local maxima and minima points from the given data. No matter for 1D signal or 2D array, neighboring window method is employed to find local maxima and local minima points. The data point/pixel is considered as a local maximum (minimum) if its value is strictly higher (lower) than all of its neighbors.

We illustrated 1-dimensional case and 2-dimensional case separately.

- 1-dimensional case:
  For each extrema map, the distance between the two neighborhood local maxima (minima, extrema, zero-crossing) has been calculated called as adjacent maxima (minima, extrema, zero-crossing) distance vector Adj_max ($Adj\_min, Adj\_ext, Adj\_zer$). Four types of window size:
  - Window-size I: $\max(Adj\_max)$;
  - Window-size II: $\max(Adj\_min)$;
  - Window-size III: $\max(Adj\_zer)$;
  - Window-size IV: $\max(Adj\_ext)$.
- 2-dimensional case:
  The window size for average filters is determined based on the maxima and minima maps obtained from a source image. For each local maximum (minimum) point, the Euclidean distance to the nearest local maximum (minimum) point is calculated, denoted as adjacent maxima (minimum) distance array Adj_max (Adj_min).
  - Window-size I: $\max(Adj\_max)$;
  - Window-size II: $\max(Adj\_min)$;

3.2 Monogenic signal

The analytic signal is the basis for all kinds of approaches which makes use of the local phase. The combination of a 1D signal and its Hilbert transform is called the analytic signal. Similarly, the combination of a image and its Riesz transform, which is the generalization of Hilbert transform, is called the monogenic signal [4, 5].

The monogenic signal is often identified as a local quantitative or qualitative measure of an image. Different approaches to an nD analytic or complex signal have been proposed in the past:

- Total Hilbert Transform, The Hilbert transform is performed with respect to both axes:

  $$H_T(\overrightarrow{v}) = j sign(v_1) sign(v_2)$$

  This transform is not a valid generalization of the Hilbert transform since it does not perform a phase shift of $\pi/2$.It can't meet orthogonality.

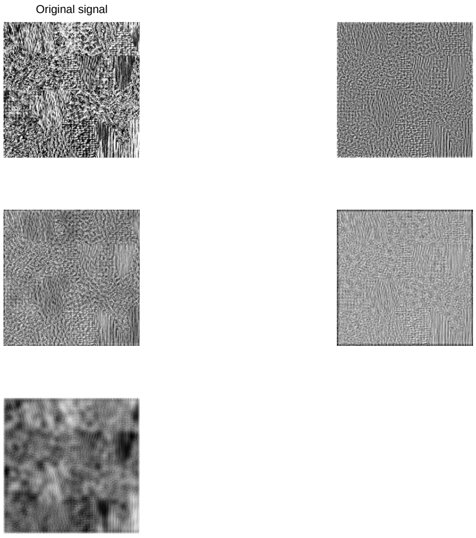

Figure 1. The 2DIMFs obtained by the improved 2DEMD.

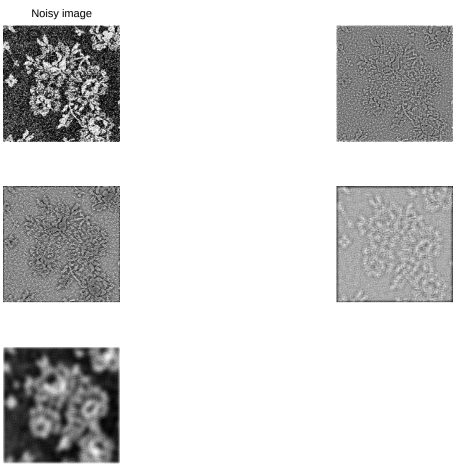

Figure 2. The 2DIMFs obtained by the improved 2DEMD on the noisy image.

- Partial Hilbert Transform, The Hilbert transform is performed with respect to a half-space that is chosen by introducing a preference direction:

  $$H_T(\overrightarrow{v}) = jsign(\overrightarrow{v}, \overrightarrow{d}),$$

  where $\overrightarrow{v} = (v_1, v_2)$ is one 2D vector, $\overrightarrow{d}$ is one preference direction. This transform missed the isotropy.

- Total Complex Signal,

- Hypercomplex Signal.

The Riesz transform [4, 5] is a multidimensional generalization of the Hilbert transform. The expression of Riesz transformed signal in the frequency domain:

$$F_R(\overrightarrow{v}) = \frac{i\overrightarrow{v}}{\overrightarrow{v}}F(\overrightarrow{v}) = H_2(\overrightarrow{v})F(\overrightarrow{v}),$$

where the transfer function H_2 of the Riesz transform is the generalization of the Hilbert transform, the corresponding space representation of Riesz transform is:

$$f_R(\overrightarrow{x}) = -\frac{\overrightarrow{x}}{2\pi|\overrightarrow{x}|^3} * f(\overrightarrow{x}) = h_2(\overrightarrow{x}) * f(\overrightarrow{x}).$$

The Riesz transformed signal and the original signal constitutes the 2D analytical signal, this is the monogenic signal.

$$f_M(\overrightarrow{x}) = f(\overrightarrow{x}) - (i, j)f_R(\overrightarrow{x}).$$

From this formulation, we see the 2D analytical signal is 3D vector and we can get the local features of the monogenic signal.

- Phase: Phase as we all know the polar representation of the complex $z = x + iy$ is $(r, \varphi) = (\sqrt{z\overline{z}}, \arg(z))$. Where $\overline{z}$ is the conjugate of z, $\arg(z)$ is the phase of the complex: $\arg(z) = a\tan 2(y, x) = sign(y)a\tan(|y|/x)$, $sign(y)$ represents the direction of rotation. The phase of the 2D analytical signal is:

  $$a\tan 3(y, x) = \frac{\overrightarrow{x}_D}{|\overrightarrow{x}_D|}a\tan(\frac{|\overrightarrow{x}_D|}{< (0,0,1)^T, \overrightarrow{x} >}),$$

  where $\overrightarrow{x}_D = (0, 0, 1)^T \times \overrightarrow{x}$ yields the direction of the rotation vector. The phase of the monogenic signal is:

  $$\varphi(\overrightarrow{x}) = a\tan 3(f_M(\overrightarrow{x})) = \arg(f_M(\overrightarrow{x})).$$

- Amplitude: The local amplitude of $f_M(\overrightarrow{x})$ is:

  $$|f_M(\overrightarrow{x})| = \sqrt{f_M(\overrightarrow{x})\overline{f_M(\overrightarrow{x})}} = \sqrt{f^2(\overrightarrow{x}) + |f_R(\overrightarrow{x})|^2},$$

  given the local phase $\varphi(\overrightarrow{x})$ and the local amplitude $|f_M(\overrightarrow{x}|$ of a monogenic signal, it can be reconstructed by

  $$f_M(\overrightarrow{x} = |f_M(\overrightarrow{x}| \exp((-j, i, 0)\varphi(x)).$$

2DEMD permits extracting multiscale components. The monogenic signal of each IMF permits to compute local amplitude, local phase and the local direction. We have shown this feature through experiment results for both natural textures and synthetic textures.

4 Experimental Results

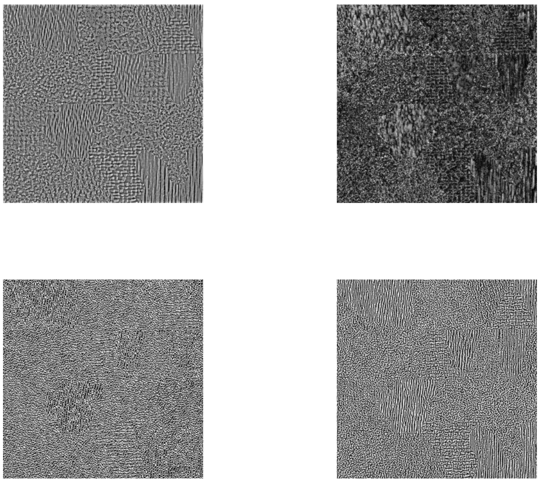

Figure 3. left-up: 1st IMF, right-up: amplitude, left-down: phase orientation, right-down: phase angle.

In all our numerical experiments we determine the window size in each decomposition with Window-size I. Unless otherwise specified we use $\alpha = 0.5$ for our stopping criterion.

We show the local features amplitude, phase orientation, phase angle extracted by SMV of the 1st IMF.

By having access to these representations of scenes or objects, we can concentrate on only one or several modes (one individual or several spatial frequency components) rather than the image entirety. The improved 2DEMD and Riesz local analysis offer a new and more promising way to analyze texture images.

5 Conclusions

This paper contains two main works. First, we proposed a new two-dimensional EMD (2DEMD) method, which is faster, better-performed than the current 2DEMD methods. Second, the Riesz transform are utilized on the 2DIMFs to

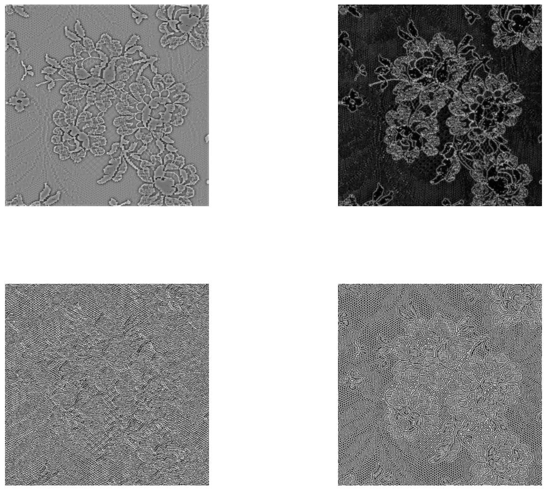

Figure 4. left-up: 1st IMF, right-up: amplitude, left-down: phase orientation, right-down: phase angle.

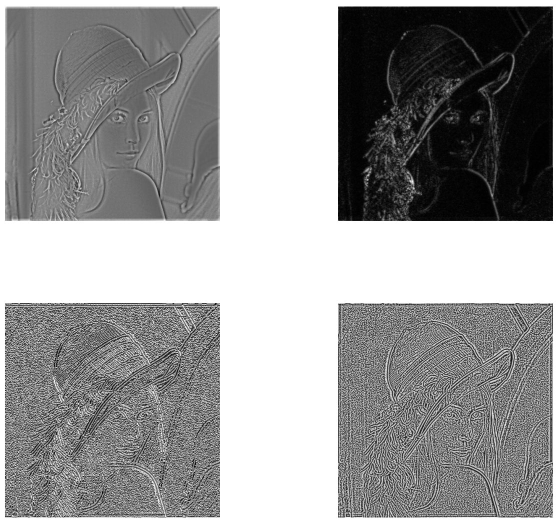

Figure 5. left-up: 1st IMF, right-up: amplitude, left-down: phase orientation, right-down: phase angle.

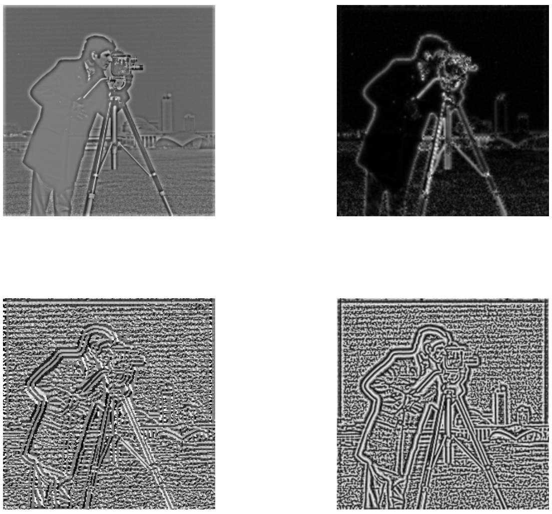

Figure 6. left-up: 1st IMF, right-up: amplitude, left-down: phase orientation, right-down: phase angle.

get the 2D analytical signals. The local features (amplitude, phase orientation, phase angle, etc) are evaluated. The performances are demonstrated on both texture images and natural images.

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