Cooperative Stochastic Differential Game in P2P Content Distribution Networks

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Abstract

Pareto-optimal resource allocation is one of the major goals in game-theoretic solutions for peer-to-peer content distribution network. These solutions have been designed that force the participants to behave more cooperatively to reach the overall efficiency as well as individual efficiency of a distributed system. But there exist many issues when these solutions are applied in stochastic differential environment which is more realistic. In this paper, we propose an incentive framework based on cooperative stochastic differential game. In this framework, peers can cooperate with each other and get Pareto-optimal bandwidth allocation without any binding agreements. We prove the payoff distribution procedure can achieve dynamic Shapley value by using equilibrating transitory compensation during peers' cooperation and make peers follow the original optimality principle and cooperative state trajectory path. Our incentive framework is more flexible and realistic than previous game-theoretic solutions. It can build cooperation relationship in stochastic dynamic peer-to-peer network without the limitation of time consistency.

1. Introduction

The Peer-to-Peer (P2P) paradigm offers obvious advantages for the fast distribution of large content in the Internet. As reported in an ipoque report [1] in 2009, P2P generates most traffic in all regions. Just as in previous years, and BitTorrent is the dominating P2P protocol followed by HTTP in all regions but South America (There is another P2P protocol which leads the pack before BitTorrent and eDonkey in South America).

Though the swarming principle supports P2P content distribution protocol in a non-cooperative environment and exploits the two-way interest of peers in different blocks which the other one provides, the cooperation of peers is not a matter of course. Most of the peers behave selfishly and are interested in maximizing their own download rates, the mutual interest results in peers, which bargain for bandwidth with each other. P2P faces the problem of free-riding [2] where peers consume resources solely without contributing anything to the network. Therefore, reputation systems and/or incentive mechanisms are implemented in P2P applications frequently.

A very popular example is the BitTorrent protocol [3] where a peer uploads to others from which it receives the highest download rates. This strategy is inspired by the tit-for-tat principle that is well known from game theory. Here, a player adopts the strategy, which his opponent used in the previous round. By cooperating in the first step the tit-for-tat strategy proved very effective in the repeated prisoner's dilemma [4].

Unfortunately, simulation-based studies for BitTorrent reveal a high variability in the download rates [5] and unfairness in terms of the ratio of uploaded to downloaded data [6]. Tit-for-tat strategy may look beneficial from a local perspective, but from a more global perspective, they are not effective. Piatek [7] shows that increased upload contribution only marginally improves download rates, and peers have no reason to contribute once they have satisfied their immediate demands.

These results lead to two questions. Firstly, from the user perspective: Does another strategy exist which outperforms BitTorrent's tit-for-tat strategy? This means with such kind of strategy a user can fairly increase its download performance according to it upload performance. Secondly, from the angle of a protocol designer: Does a strategy exist which ensures fairness between peers although peers behave selfishly?

Users in a P2P social network are strategic and rational, and they are likely to manipulate any incentive systems to maximize their own payoff. They will even cheat if they believe it could help maximize their payoff. Hence, game theory is a proper tool to model the interaction among peers, and to analyze the optimal and cheat-proof cooperation strategies. But there exist many issues when the proposed solutions are applied in stochastic differential environment which is more realistic.

Our work tries to answer the above two questions based on game theory. We design an incentive framework based on cooperative stochastic differential game to achieve Pareto-optimal bandwidth allocation fairly without any binding agreements. Our contributions are:

- We analyze the root issue of BitTorrent tit-for-tat strategy from game theory prospective and reveal the reason that why this strategy cannot achieve its original goal.
- We clearly define the basic elements of game theory from P2P content distribution network's prospective, which lay a solid foundation to help us explore the essence of incentive framework based on game theory in P2P content distribution network.
- We present a general incentive framework based on cooperative stochastic differential game. It adopts an analytically tractable "payoff distribution procedure" which would lead to subgame-consistent solutions. We prove the payoff distribution procedure can achieve dynamic Shapley value by using equilibrating transitory compensation during peers' cooperation and make peers follow the original optimality principle and cooperative state trajectory path.

1.1 Related work

We now briefly present some related work. Micro-payment [8] is probably the earliest work on designing incentive protocol for P2P networks. It relies on a centralized server and uses virtual currency to provide incentive for resource sharing. Since then, much efforts are focused on incentive mechanisms for P2P systems [9] and wireless networks [10]. Vishnumurthy [11] shows that shared history based incentives can overcome the scalability problem of private history based mechanisms and one can use DHT to implement the shared history incentive mechanism. One shared history based incentive is the reciprocative strategy [12]. It makes decisions according to the reputation of requesters and is studied via simulation only.

There are some existing works on designing a particular incentive mechanism. Feldman [13] assumes that each peer has a fixed strategy with a certain distribution while we assume peers can adapt their strategies. In [14], authors show that a proportional strategy can lead to market equilibria but the result does not generalize to multiple strategies.

Pareto-optimal resource allocation is one of the major goals in game-theoretic solutions for peer-to-peer content distribution network. The above solutions have been designed that force the participants to behave more cooperatively to reach the overall efficiency as well as individual efficiency of a distributed system. But there exist many issues when these solutions are applied in stochastic differential environment which is more realistic. Our paper focuses on the incentive mechanism design in cooperative stochastic differential environment in P2P content distribution systems.

1.2 Paper organization

The balance of the paper is organized as follows. In section 2, we analyze the root issue of tit-for-tat strategy and reveal the reason that why this strategy cannot achieve its original goal. In section 3, we clearly define the basic elements and their properties of game theory from P2P content distribution network's view. In section 4, we present a general incentive framework based on cooperative stochastic differential game and prove its effectiveness. Finally, section 5 concludes this paper.

2. Analysis of Tit-for-Tat Strategy

From section 1, we know that the swarming principle supports P2P content distribution protocol in a non-cooperative environment. But under non-cooperative environment, the profit of each peer and whole system will be far away from Pareto-optimality, even hurting each other. The result must be "solitary, poor, nasty, brutish and shot" [15]. The difficulty of cooperation is rooted from free-riding problem in many situations. As a matter of fact, the reason is the temporary profit of each peer exits conflicts.

Based on this analysis, many researchers induct game theory as tool to solve this problem. Because static non-cooperative game often leads to lose-lose situation, so the static cooperative game which can build Pareto-optimality is applied in P2P systems. But there is an assumption in static cooperative game that peers can meet a binding agreement. Actually, whether there is a binding agreement is the watershed between non-cooperative game and cooperative game. The Shapley value in cooperative game is a very important solution which can create a unique fairly profit distribution which can achieve Pareto-optimality. Unfortunately, binding agreement has not one hundred percent sanction in real P2P systems. There always exist conflicts between individual rationality and group rationality.

Under this circumstance, strategic conflict resolution becomes a way of building cooperation in non-cooperative P2P environment. We realize that repeated game still can bring on Tragedy of Common, but there can exist cooperation in infinite repeated game. The most used strategy is tit-for-tat which is implemented in BitTorrent protocol. Here we should be noted that infinite repeated game is a very special game model which often does not exist in real P2P systems. That is why the tit-for-tat strategy often cannot achieve fairness bandwidth allocation.

On one hand, since peers interact in time and decisions generally lead to effects over time, it is only a slight exaggeration to claim that "P2P content distribution is a dynamic game". Dynamic or differential game investigates interactive decision making over time. On the other hand, the dynamic cooperation among pees often changes stochastically. For example, there are unstable network bandwidth and unexpected lost connection among peers, etc. Yeung [16] introduced the paradigm of randomly-furcating stochastic differential games to make it possible to study stochastic elements via branching payoffs under the format of differential games.

Cooperative games hold out the promise of socially optimal and group efficient solutions to problems involving strategic actions. Formulation of optimal peer behavior is a fundamental element. In dynamic cooperative games, a stringent condition on cooperation and agreement is required: In the solution, the optimality principle must remain optimal throughout the game, at any instant of time along the optimal state trajectory determined at the outset. This condition is known as dynamic stability or time consistency. In other words, dynamic stability of solutions to any cooperative differential game involved the property that, as the game proceeds along an optimal trajectory, players are guided by the same optimality principle at each instant of time, and hence do not possess incentives to deviate from the previously adopted optimal behavior throughout the game.

In the presence of stochastic elements, a more stringent condition, subgame consistency, is required for a credible cooperative solution. In particular, a cooperative solution is subgame-consistent if an extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behavior would remain optimal. Along with Yeung and Petrosyan's significant breakthrough in the study of cooperative stochastic differential games [17], we propose an incentive framework based on their contributions and apply it in P2P content distribution networks.

Before we introduce our incentive framework, we need to define the basic elements of game theory from P2P content distribution network's prospective to lay a solid foundation and help us explore the essence of incentive framework based on game theory.

3. Basic Elements of Game Theory

In P2P content distribution environment, we need provide clear definition of the following basic elements in game theory: player, action, information, strategy, payoff, rationality, objective, order of play, outcome and equilibrium.

Definition 1. *Player* is peer which can make decisions in P2P content distribution.

Player must have more than one action to be selected. Normally, there are at least two peers in P2P content distribution. Except player, there can exist pseudo-player, namely, nature. *Nature* is a player or mechanism without objective. It will randomly choose action as pre-defined probability.

In general, player is assumed as rational.

Definition 2: *Action* is decision variable of player in a specific time point. We use $a_i \in A_i$ denotes a specific action of i-th player. A_i denotes the action sets that the player can select.

The action of player can be discrete or continuous. In P2P content distribution networks, we can view it as the bandwidth that a peer wants to upload.

Definition 3: *Information* is knowledge of peer in P2P content distribution game. We use information set to describe information. In game theory, an *information set* is a set that, for a particular player, establishes all the possible moves that could have taken place in the game so far, given what that player has observed so far.

If the game has *perfect information*, every information set contains only one member, namely the point actually reached at that stage of the game. Otherwise, it is *imperfect information* that some players cannot be sure exactly what has taken place so far in the game and what their position is.

An item of information in a game is *common knowledge* if all of the players know it (it is mutual knowledge) and all of the players know that all other players know it and all other players know that all other players know that all other players know it, and so on.

Complete information requires that every player know the strategies and payoffs of the other players but not necessarily the actions. If and only if the type, strategy space and payoff function of each peer are all common knowledge, we call the game has complete information, otherwise, it has incomplete information. We assume the information in a game is complete. If not, we can use Harsanyi Transformation which adds *nature* as a player in the game to make an incomplete information game transform as a complete but imperfect information game, or Bayesian game.

Definition 4: A player's *strategy* in a game is a complete plan of action for whatever situation might arise; this fully determines the player's behavior. A player's strategy will determine the action the player will take at any stage of the game, for every possible history of play up to that stage.

There are two kinds of strategies: pure strategy and mixed strategy. A pure strategy provides a complete definition of how a player will play a game. A mixed strategy is an assignment of a probability to each pure strategy. This allows for a player to randomly select a pure strategy.

We use $s_i \in S_i$ denotes a specific strategy of i-th player. S_i denotes the strategy set or strategy space.

Definition 5: *Payoffs* are numbers which represent the motivations of players. Payoffs may represent profit, quantity, "utility," or other continuous measures, or may simply rank the desirability of outcomes. In P2P content distribution networks, payoff of a peer is the download bandwidth of a peer. We use u_i denotes payoff of i-th player.

Definition 6: *Rationality* implies that every player is motivated by maximizing his own payoff. In a stricter sense, it implies that every player always maximizes his utility, thus being able to perfectly calculate the probabilistic result of every action. Rationality can be classified as individual rationality and group rationality.

Definition 7: If each player's preference can be represented as expected payoff function, then player has a clear and definite *objective* that optimizes its payoff function by strategy or choosing actions.

Definition 8: In game theory, each player has time point to select action. These time points are called decision nodes. Decision nodes can have sequence which is called as *order of play*. In differential game, players can take actions simultaneously or with sequential order.

Definition 9: *Outcome* is a set of moves or strategies taken by the players, or their payoffs resulting from the actions or strategies taken by all players.

Definition 10: *Equilibrium* is the combination of optimal strategies among all players. In an equilibrium, each player of the game has adopted a strategy that they are unlikely to change. We use s_i^* denotes optimal strategy of i-th player.

4. Cooperative Stochastic Differential Game

4.1 Incentive Framework

The balance of the According to Yeung [16] and [17], we know that the dynamic stability of a solution of a cooperative differential game is the property that, when the game proceeds along an optimal trajectory, at each instant of time the players are guided by the same optimality principles, and hence do not have any ground for deviation from the previously adopted "optimal" behavior throughout the game.

In the presence of stochastic elements, a more stringent condition, subgame consistency, is required for a credible cooperative solution. A cooperative solution is subgame consistent if an extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behaviors would remain optimal.

Consider the cooperative game $\Gamma_c(x_0, T-t_0)$ in which the players agree to act to maximize their joint payoff and adopt a certain mechanism governing the sharing of the players' payoffs. t_0 and x_0 are start time and start state. To achieve group rationality, the players adopt the cooperative controls $[\psi_1^{(t_0)*}(t,x), \psi_2^{(t_0)*}(t,x)]$. A set of controls $[\psi_1^{(t_0)*}(t,x), \psi_2^{(t_0)*}(t,x)]$ provides an optimal solution to the stochastic control problem $\Psi(x_0, T-t_0)$. The optimal cooperative state trajectory follows the stochastic path $\{x^*(s)\}_{s=t_0}^{T}$.

At time t_0, with the state being x_0, the term $\xi^{(t_0)i}(t_0, x_0)$ denotes the expected share/imputation of total cooperative payoff (received over the time interval $[t_0, T]$) to player i guided by the agreed-upon optimality principle.

Now, consider the cooperative game $\Gamma_c(x_\tau^*, T-\tau)$ in which the game starts at time $\tau \in [t_0, T]$ with initial state $x_\tau^* \in X_\tau^*$, and the same agreed-upon optimality principle as above is adopted. Let $\xi^{(\tau)i}(\tau, x_\tau^*)$ denote the expected share/imputation of total cooperative payoff given to player i over the time interval $[\tau, T]$.

Following Yeung and Petrosyan [17], we formulate a payoff distribution over time so that the agreed imputations can be realized. Let the vectors $B^\tau(s) = [B_1^\tau(s), B_2^\tau(s)]$ denote the instantaneous payoff of the cooperative game at time $s \in [\tau, T]$ for the cooperative game $\Gamma_c(x_\tau^*, T-\tau)$. In other words, player i, for $i \in \{1, 2\}$, obtains a payoff equaling $B_i^\tau(s)$ at time instant s. A terminal value of $q^i(x^*(T))$ is received by player i at time T.

In particular, $B_i^\tau(s)$ and $q^i(x^*(T))$ constitute a Payoff Distribution Procedure (PDP) for the game $\Gamma_c(x_\tau^*, T-\tau)$. $B_i^\tau(s)$ is equilibrating transitory compensation, $q^i(x^*(T))$ is optimal terminal value. Subgame consistency guarantees that the solution imputations throughout the game interval in the sense that the extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behaviors would remain optimal. Equilibrating transitory compensation is developed for the implementation of subgame consistency cooperation scheme.

In stochastic environment, the compensation $B_i(\tau)$ player i receives at time τ given the state $x_\tau^* \in X_\tau^*$ is the sum of

(*i*) Player i's agreed upon marginal share of total expected cooperative profit,

(*ii*) Player i's agreed upon marginal share of his own expected non-cooperative profit plus the instantaneous effect on his non-cooperative expected payoff when the change in the state variable x_τ^* follows the cooperative trajectory instead of the non-cooperative path, and

(*iii*) Player i's agreed upon marginal share of Player j's non-cooperative profit plus the instantaneous effect on Player j's non-cooperative payoff when the change in the state variable x follows the optimal trajectory instead of the non-cooperative path.

In P2P content distribution networks, consider the scenario in which n peers, and peer i's objective is:

$$E_{t_0}\{\int_{t_0}^{T} g^i[s, x_i(s), u_i(s)]\exp[-\int_{t_0}^{s} r(y)dy]ds + \exp[-\int_{t_0}^{s} r(y)dy]q^i(x_i(T))\} \tag{1}$$

for

for $i \in [1, 2, \ \ldots, \ n] \equiv N$, $g^i(\bullet) \geq 0, q^i(\bullet) \geq 0$

where $x_i(s) \in X_i$ denotes the current bandwidth of peer i, $u_i \in U^i$ is the control vector of peer i, denotes the upload bandwidth of peer i can provide, $\exp[-\int_{t_0}^{s} r(y)dy]$ is discount factor which can be viewed as a peer's opportunity cost, and $q^i(x_i(T))$ is the terminal payoff which can be viewed as the future potential current value (bandwidth) in terminal time. $g^i[s, x_i(s), u_i(s)]$ denotes the instantaneous bandwidth that peer i can get. In particular, $g^i[s, x_i(s), u_i(s)]$ and $q^i(x_i)$ are positively related to x_i.

The state dynamics of the game is characterized by the set of vector-valued stochastic differential equations:

$$dx_i(s) = f^i[s, x_i(s), u_i(s)]ds + \sigma[s, x_i(s)]dz_i(s), \quad x_i(t_0) = x_i^0 \tag{2}$$

where $\sigma[s, x_i(s)]$ is a $m_i \times \Theta_i$ matrix, and $z_i(s)$ is a Θ_i -dimensional Wiener process and the initial state x_i^0 is given. Let $\Omega_i[s, x_i(s)] = \sigma[s, x_i(s)]\sigma[s, x_i(s)]^T$ denote the covariance matrix with its element in row h and column ζ denoted by $\Omega_i^{h\zeta}[s, x_i(s)]$. For $i \neq j$, $x_i \cap x_j = \varnothing$, and $z_i(s)$ and $z_j(s)$ are independent Wiener processes. Trough the above processes, we import stochastic environment factors in dynamic cooperation joined by multiple peers. We also used $x_N(s)$ to denote the vector $[x_1(s), x_2(s), \ . . . , x_n(s)]$ and x_N^0 denotes the vector $[x_1^0, x_2^0, \ldots, x_n^0]$.

Consider a coalition of a subset of peers $K \subseteq N$. There are k peers in the subset K. The participating peers can gain more bandwidth that would be difficult for them to obtain on their own, and hence the state dynamics of peer i in the coalition K becomes

$$dx_i(s) = f^i[s, x_K(s), u_i(s)]ds + \sigma[s, x_i(s)]dz_i(s), \quad x_i(t_0) = x_i^0 \quad \text{for } i \in K \tag{3}$$

where $x_K(s)$ is the concatenation of the vectors $x_j(s)$ for $j \in K$. In particular, $\partial f_i^K[s, x_K, u_i]/\partial x_j \geq 0$, for $j \neq i$. Thus positive effects on the state of peer i could be derived from the other peers within the coalition. Without much loss of generalization, the effect of x_j on $f_i^K[s, x_K, u_i]$ remains the same for all possible coalitions K containing peers i and j.

4.2 The Dynamic Shapley Value Imputation

Consider the above cooperation involving n peers. The member peers would maximize their joint profit and share their cooperative profits according to the Shapley value. The Shapley value is one of the most commonly used sharing mechanism in static cooperation games with transferable payoffs. Besides being individually rational and group rational, the Shapley value is also unique. The uniqueness property makes a more desirable cooperative solution relative to other solutions like the Core or the Stable Set. Specifically, the Shapley value gives an imputation rule for sharing the cooperative profit among the members in a coalition as:

$$\varphi^i(v) = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!}(v(K) - v(K \setminus i)) \quad (4)$$

$$\text{for } i \in N$$

where $K\backslash i$ is the relative complement of i in K, $v(K)$ is the profit of coalition K, and $[v(K) - i(K\backslash i)]$ is the marginal contribution of peer i to the coalition K.

Given the assumption that $v(K)$ is super-additive, the Shapley value yields the desirable properties of individual rationality and group optimality. Though the Shapley value is used as the profit allocation mechanism, there exist two features that do not conform with the standard Shapley value analysis. The first is that the present analysis is dynamic so that instead of a one-time allocation of the Shapley value, we have to consider the maintenance of the Shapley value imputation over the cooperation horizon. The second is that the profit $v(K)$ is the maximized profit to coalition K, and is not a characteristic function (from the game in which coalition K is playing a zero-sum game against coalition $N\backslash K$).

To maximize the cooperation's profits the peers would adopt the control vector $\{\psi_N^{(t_0)N*}(t, x_N^{t*})\}_{t=t_0}^T$ over the time $[t_0, T]$ interval, and the corresponding optimal state trajectory $\{x_N^{t*}(t)\}_{t=t_0}^T$ would result. At time t_0 with state $x_N^{t_0}$, the peers agree that peer i's share of profits be:

$$v^{(t_0)i(t_0,x_N^0)} = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!}[W^{(t_0)K}(t_0, x_K^0) - W^{(t_0)K\setminus i}(t_0, x_{K\setminus i}^0)] \quad (5)$$

$$\text{for } i \in N$$

However, the Shapley value has to be maintained throughout the cooperation horizon $[t_0, T]$. In particular, at time $\tau \in [t_0, T]$ with the state being $x_N^{\tau*}$, the following imputation principle has to be maintained:

Condition 1. At time τ, peer i's share of profits be:

$$v^{(\tau)i(\tau,x_N^{\tau*})} = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!}[W^{(\tau)K}(\tau, x_K^{\tau*}) - W^{(\tau)K\setminus i}(\tau, x_{K\setminus i}^{\tau*})] \quad (6)$$

$$\text{for } i \in N \text{ and } \tau \in [t_0, T]$$

Condition 1 satisfies the property of Pareto optimality throughout the game interval and guarantees individual rationality throughout the game interval. Pareto optimality and individual rationality are essential properties of imputation vectors. Moreover, if Condition 1 can be maintained, the solution optimality principle – sharing profits according to the Shapley value – is in effect at any instant of time throughout the game along the optimal state trajectory chosen at the outset. Hence time consistency is satisfied and no peers would have any incentive to depart the cooperation. Therefore a dynamic imputation principle leading to formula (6) is dynamically stable or time consistent.

Crucial to the analysis is the formulation of a profit distribution mechanism that would lead to the realization of Condition 1. This will be done in the next section.

4.3 The Dynamic Shapley Value Imputation

In this section, a profit distribution mechanism will be developed to compensate transitory changes so that the Shapley value principle could be maintained throughout the venture horizon. From Yeung [16] and [17], we can get the following theorem.

Theorem 1. A payment to peer $i \in N$ at time $\tau \in [t_0, T]$ equaling

$$\begin{aligned} B_i(\tau) = \\ & -\sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!}\{[W_t^{(\tau)K}(\tau, x_K^{\tau*})\big|_{t=\tau}] - [W_t^{(\tau)K\setminus i}(\tau, x_{K\setminus i}^{\tau*})\big|_{t=\tau}] \\ & +([W_{x_N^{\tau*}}^{(\tau)K}(t, x_K^{\tau*})\big|_{t=\tau}] - [W_{x_N^{\tau*}}^{(\tau)K\setminus i}(\tau, x_{K\setminus i}^{\tau*})\big|_{t=\tau}]) \\ & \times f^N[\tau, x_N^{\tau*}, \psi_N^{(\tau)N}(\tau, x_N^{\tau*})]\} \end{aligned} \quad (7)$$

will lead to the realization of the Condition 1.

Since the partial derivative of $W^{(\tau)K}(\tau, x_K^{\tau*})$ with respect to x_j, where $j \notin K$, will vanish, a more concise form of Theorem 1 can be obtained as:

Theorem 2. A payment to peer $i \in N$ at time $\tau \in [t_0, T]$ leading to the realization of the Condition 1 can be expressed as:

$$\begin{aligned} B_i(\tau) = \\ & -\sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!}\{[W_t^{(\tau)K}(\tau, x_K^{\tau*})\big|_{t=\tau}] - [W_t^{(\tau)K\setminus i}(\tau, x_{K\setminus i}^{\tau*})\big|_{t=\tau}] \\ & +\sum_{j \in K}[W_{x_j^{\tau*}}^{(\tau)K}(t, x_K^{\tau*})\big|_{t=\tau}] f_j^N[\tau, x_N^{\tau*}, \psi_N^{(\tau)N}(\tau, x_N^{\tau*})] \\ & -\sum_{h \in K\setminus i}[W_{x_h^{\tau*}}^{(\tau)K\setminus i}(\tau, x_{K\setminus i}^{\tau*})\big|_{t=\tau}] f_h^N[\tau, x_N^{\tau*}, \psi_h^{(\tau)N}(\tau, x_N^{\tau*})]\} = \\ & \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!}\{[W_t^{(\tau)K}(\tau, x_K^{\tau*})\big|_{t=\tau}] - [W_t^{(\tau)K\setminus i}(\tau, x_{K\setminus i}^{\tau*})\big|_{t=\tau}] \\ & +[W_{x_N^{\tau*}}^{(\tau)K}(t, x_K^{\tau*})\big|_{t=\tau}] f_K^N[\tau, x_N^{\tau*}, \psi_K^{(\tau)N}(\tau, x_N^{\tau*})] \\ & -[W_{x_{K\setminus i}^{\tau*}}^{(\tau)K\setminus i}(t, x_{K\setminus i}^{\tau*})\big|_{t=\tau}] f_{K\setminus i}^N[\tau, x_N^{\tau*}, \psi_{K\setminus i}^{(\tau)N}(\tau, x_N^{\tau*})] \end{aligned},$$

where $f_K^N[\tau, x_N^{\tau*}, \psi_K^{(\tau)N}(\tau, x_N^{\tau*})]$ is a column vector containing $f_i^N[\tau, x_N^{\tau*}, \psi_i^{(\tau)N}(\tau, x_N^{\tau*})]$ for $i \in K$.

The vector $B(\tau)$ serves as a form equilibrating transitory compensation that guarantees the realization of the Shapley value imputation throughout the game horizon. Note that the instantaneous profit $B_i(\tau)$ offered to peer i at time τ is conditional upon the current state $x_N^{\tau*}$ and current time τ. One can elect to express $B_i(\tau)$ as $B_i(\tau, x_N^{\tau*})$. Hence an instantaneous payment $B_i(\tau, x_N^{\tau*})$ to player $i \in N$ yields a dynamically stable solution to the cooperation.

5. Conclusion

In this paper, we analyze the root of issues derived from BitTorrent tit-for-tat strategy, reveal that the cooperative stochastic differential environment is more realistic than the current proposed mechanism. Based on the clear definition of basic game theory elements from P2P content distribution prospective, we propose an incentive framework which peers can cooperate with each other and get Pareto-optimal bandwidth allocation without any binding agreements. We prove the payoff distribution procedure can achieve dynamic Shapley value by using equilibrating transitory compensation during peers' cooperation and make peers follow the original optimality principle and cooperative state trajectory path. Our incentive framework is more flexible and realistic than previous game-theoretic solutions. It can build cooperation relationship in stochastic dynamic peer-to-peer network without the limitation of time consistency.

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