Minority Games and Distributed Coordination in Non-Stationary Environment

Presented by: Tingting Wang

Based on a paper with the same name authored by Aram Galstyan and Kristina Lerman, available at:
http://www.isi.edu/~lerman/papers/mg-conf.pdf
Minority Game (MG) – Introduced by Challet and Zhang in 1997, is a simple game with artificial agents with *partial information* and *bounded rationality*. It captures the essential feature of systems where agents compete for limited resources, like financial markets.

Features of Standard MG:
- There are N(odd) players
- At each iteration they choose a side – A or B (0 or 1)
- Players earn one point if they are in the minority
- Each player knows what was the result of $M$ previous time steps (the history)
- Each player has $S$ strategies
- Model is completely defined given N, M, S
Branches of MG

- capacity level $\eta = 1/2$:
  - Standard MG: with $N$ agents, each has $M$ memory sizes and $S$ strategies. 2 choices
  - Evolutionary MG: has only 1 strategy $S$, each agent $i$ has a probability $p_i$ to choose using the strategy $S$ or not. *Evolutionary* means that if an agent has a wealth smaller that $d$, his $p_i$ is changed within a range of $R$. 2 choices
  - Multi-choices MG: number of choices $> 2$. Other features = Standard MG
  - Hierarchical MG: MG using local histories, and integrated by higher level MG.

- Capacity level $0 < \eta < 1$
  - Generalized MG

In the above 2 categories, environments are stable.

- Propose of non-stationary environments:
  - By Aram Galstyan and Kristina Lerman at 2002
  - MG with arbitrary capacities: $\eta(t) = \eta_0 + \eta_1(t)$
    The winning choice is “1” if $A(t) \leq L$ where $L$ is the capacity, $A(t)$ is the number of agents that chose “1”
Model of Non-stationary Environments
(Kauffman Networks)

- A set of $N$ boolean agents, choose between 0 and 1: $s_i = \{0, 1\}, i = 1, ..., N.$

$$s_i(t + 1) = \mathbf{F}_i^j(s_{k_1}(t), s_{k_2}(t), ..., s_{k_K}(t))$$

where $s_{ki}, i = 1, ..., K$ are the set of neighbors

- Strategy: $F_i^j, j = 1, ... S$, are called a strategies, which are a set of $S$ randomly chosen boolean functions used by agent $i$, and the score of $F_i^j$ at time step $t$ is $U_i^j(t)$.

- Capacity Level: $\eta(t) = \eta_0 + \eta_1(t)$

- Attendance: $A(t) = \sum_{i=1}^{N} s_i(t)$

so, if $A(t) \leq N\eta(t)$, the winning choice is “1”, and “0” otherwise.
Model of Non-stationary Environments (cont.)

- $\delta(t) = A(t) - N\eta(t)$, which describes the standard deviation from the optimal resource utilization.

- The Global measure for optimality $\sigma^2$ is defined as:

$$\sigma^2 = \frac{1}{T_0} \sum_{t=t_0}^{t_0+T_0} \delta(t)^2$$

when $\eta_1(t)=0$, this quantity is the squared standard deviation in traditional MG.

- In the following experiments, the parameters are:
  - N: 100~5000
  - S = 2, which is chosen randomly from $2^k$ possible boolean functions.
  - Using different forms of $\eta(t)$.
A segment of the attendance time series for $\eta(t) = 0.5 + 0.15\sin(2\pi t/T)$, $T=1000$ and different network connectivity $K$.

- $K<2$: the network reaches a frozen configuration
- $K=2$: networks show a tendency towards self-organization into a coordinated phase characterized by small fluctuations and effective resource utilization
- $K>2$: the dynamics of the system is chaotic.
Experimental Results -- II

- Use the model of traditional MG with M=6, which corresponding to the minimum of $\sigma$.
- Use the same $\eta(t)$ as in the last experiment.
- Results:
  - The system reacts to the external change.
  - The overall performance in terms of resource allocation as described by $\sigma$ is much poorer.
  - The distribution of wealth among the player is much wider than in the system with local information exchange---more fair.
Coordination occurs even in the presence of vastly different time scales in the environmental dynamics.
Experimental Results -- IV

- The variance reaches minimum value when $K=2$, and is independent of the number of agents in the system $\sigma^2/N \approx \text{const}$ different with traditional MG.
- When $K$ increases, the variance is tend to flat and depends on the amplitude of the perturbation and the number of agents in the system.
- When $K=2$, $\sigma \propto N$; for others, $\sigma \propto N^{1/2}$.
Phase Transitions in Kauffman Nets

Kauffman Nets: phase transition at $K=2$ separating ordered ($K<2$) and chaotic ($K>2$) phases

For $K>2$ one can arrive at the phase transition by tuning the homogeneity parameter $P$ (the fraction of 0’s or 1’s in the output of the Boolean functions).

The coordinated phase might be related to the phase transition in Kauffman Nets.

$K=3$\[ P_c \approx 0.78 \]
Summary of Results

- Generalized Minority Games on K=2 Kauffman Nets are highly adaptive and can serve as a mechanism for distributed resource allocation
  - In the coordinated phase the system is highly scalable
  - The adaptation occurs even in the presence of different time scales, and without the agents explicitly coordinating or knowing the resource capacity
  - For K>2 similar coordination emerges near the phase transitions point of the ordered/chaotic phase in the corresponding Kauffman Networks
Problems

- Lack of sufficient experiments on other important factors of MG, for example, the number of strategies $S = 2$, how about other number of $S$?

- The author compares the performance of Kauffman MG model with $K=2$ with standard MG model with $M = 6$. Actually $K$ can be mapped to $M$, so why not compare with $M = 2$?
  (The assumption of MG is, the agents only know about the global signal, however in this Kauffman MG model, the strategy is based on other input of its neighbors. I think this is a very large difference, so the comparison with standard MG is improper)

- Only the periodic perturbations are used to change the capacity level: $\eta(t) = 0.5 + 0.15 \sin(2\pi t/T)$. How about the random disturbance or other forms of distributions to the capacity level? For example, Gaussian Distribution.

- In our search, can we can use the evaluation function to adjust the capacity level? This attempt should be carried out in 2 steps:
  - Learn the function between the evaluation of RSL and the specific problem we want to address, e.g., the real-time roles distribution in terms of the evaluation of RSL.
  - Use the real-time roles distribution as the capacity function in Kauffman MG model
Appendix: Kauffman Network

Consider a network of $N$ agents where each agent is assigned a Boolean variable $\sigma_i = 0$ or $1$. Each agent receives input from $K$ other distinct agents chosen at random in the system. The set of inputs for each agent $i$ is quenched. The evolution of the system is specified by $N$ Boolean functions of $K$ variables, each of the form

$$\sigma_i(t + 1) = f_i[\sigma_{i_1}(t), \sigma_{i_2}(t), \ldots \sigma_{i_K}(t)].$$  \hspace{1cm} (1)

There exist $2^{2^K}$ possible Boolean functions of $K$ variables. Each function is a lookup table which specifies the binary output for a given set of binary inputs. In the simplest case defined by Kauffman, where the networks do not organize, each function $f_i$ is chosen randomly among these $2^{2^K}$ possible functions with no bias; we refer to this case as the random Kauffman network (RKN).