Joint Access Point Placement and Power-Channel-Resource-Unit Assignment for 802.11ax-Based Dense WiFi with QoS Requirements

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***Abstract*—IEEE 802.11ax is a promising standard for the next-generation WiFi network, which uses orthogonal frequency division multiple access (OFDMA) to segregate the wireless spectrum into time-frequency resource units (RUs). In this paper, we aim at designing an 802.11ax-based dense WiFi network to provide WiFi services to a large number of users within a given area with the following objectives: (1) to minimize the number of access points (APs); (2) to fulfil the users' throughput requirement; and (3) to be resistant to AP failures. We formulate the above into a joint AP placement and power-channel-RU assignment optimization problem, which is NP-hard. To tackle this problem, we first derive an analytical model to estimate each user's throughput under the mechanism of OFDMA and a widely used interference model. We then design a heuristic algorithm to find high-quality solutions with polynomial time complexity. Simulation results show that our algorithm can achieve the optimal performance for a small area of 50 $\times$ 50 m^2. For a larger area of 100 $\times$ 80 m^2 where we cannot find the optimal solution through an exhaustive search, our algorithm can reduce the number of APs by 32 $\sim$ 55% as compared to the random and Greedy solutions.**

***Index Terms*—IEEE 802.11ax, AP placement, quality of service, fault tolerance, resource assignment, dense WiFi network**

I. Introduction

The IEEE 802.11ax-based dense WiFi network has attracted more and more attention from the industry and academia [1]. In dense WiFi scenarios, many users are gathered in a region, which creates a great demand on WiFi services, like upload/download videos to/from the network. In this case, many access points (APs) are required and the distance between adjacent APs is usually very close [1]. In a traditional 802.11-based dense WiFi setup, sufficient bandwidth does not necessarily translate into high throughput [2]. To improve the user experience in dense WiFi scenarios, 802.11ax [3], which acts as the next generation WiFi standard, has been investigated. It supports orthogonal frequency division multiple access (OFDMA), in which subcarriers in a channel are divided into groups which are called ***resource units (RUs)*** [3]. By strategically assigning RUs to stations, stations can transmit data simultaneously. In addition, it supports both the 2.4 and 5 GHz bands, which means that we have more non-overlapping channels to choose from to reduce the interference between neighboring APs. In short, deploying 802.11ax-based dense WiFi network is both crucial and urgent. There are two main factors that affect the network performance. The first one is the AP placement. The second one is resource (such as power, channel, and RU, etc.) assignment for the APs/stations. Furthermore, users demand continuous WiFi services even under AP's failures [4]. Unfortunately, there is little research on joint AP placement and resource assignment for 802.11ax-based dense WiFi network with quality of service (QoS) guarantees. This is why we try to address this problem.

We consider a given region with many potential users at known locations (e.g., in a stadium, each spectator is assigned a fixed seat). Our problem can be described as follows. Given a set of AP possible candidate locations and a set of stations with known locations, find out the minimum number of APs and their locations, under the joint design of AP placement and power-channel-RU assignment, to deploy an 802.11ax-based dense WiFi network that fulfils the following two QoS requirements: 1) Fault tolerance requirement. That is, when n ($n = 0, 1, 2, ...$) APs fail simultaneously, stations associated with the failed APs can still re-associate with the remaining APs to obtain the WiFi service. 2) ***User satisfaction ratio (USR)*** requirement, which is a new concept introduced in our paper. That is, we ensure that the throughput of at least β % ($0 \leq \beta \leq 100$) of the stations is no less than ρ_H, and the throughput of $(100 - \beta)$ % of the stations is no less than ρ_L, where ρ_L and ρ_H represent two throughput thresholds which can be obtained beforehand by historical data with $\rho_L < \rho_H$. We assume that ρ_L is the minimum throughput that users can accept while ρ_H is the throughput that users are satisfied with. The reason why we introduce the user satisfaction ratio requirement is that in dense WiFi scenarios, it is expensive to satisfy all users. In fact, the locations of some stations may be far away from the existing APs. If we have to satisfy them, we may need to add many APs to the network which is not cost effective. But instead, we just ensure that the throughput of the far-away stations is no less than ρ_L. This approach not

TABLE I
DIFFERENCES BETWEEN OUR WORK AND THE EXISTING WORKS

Work	Any n APs Fault Tolerance?	Satisfy USR Requirement?	For Dense WiFi?	Joint AP Placement and Power-Channel-RU Assignment?	Support 802.11ax?	Minimize Cost?	Remarks
Our work	Yes	Yes	Yes	Yes	Yes	Yes	Minimize the number of APs
[4]	No	No	No	No	No	Yes	Minimize the number of APs
[19]	No	No	No	No	No	No	Maximize the system throughput
[20]	No	No	No	No	No	Yes	Minimize the number of APs
[21]	No	No	No	No	No	Yes	Minimize the number of APs
[22]	No	No	No	No	No	Yes	Minimize the total cost of all APs
[23]	No	No	No	No	No	No	Maximize the coverage
[24]	No	No	No	No	No	No	Maximize the coverage
[25]	No	No	No	No	No	No	Minimize total transmission power of APs
[26]	Yes	No	Yes	No	No	No	Re-allocate resources when n APs fail
[27]	No	No	No	No	No	No	Re-allocate resources when one AP fails

only meets the basic needs but also saves cost in providing the WiFi service. Hence, the contributions of our research include the followings.

1) **New Problem:** We deploy an IEEE 802.11ax-based dense WiFi network in a given region with many potential users by jointly optimizing AP placement and power-channel-RU assignment. We formulate an optimization problem that minimizes the number of APs subject to the fault tolerance constraints and the user satisfaction ratio requirements. As the sub-problems of our problem are NP-hard, our problem is therefore NP-hard as well. The main differences between our work and the existing ones are summarized in Table I, which shows that our problem is quite different from the others.

2) **New Solution:** According to the mechanism of OFDMA and a widely used interference model, we design efficient power adjustment, channel assignment, and RU assignment methods, based on which we derive the throughput of the stations. We propose a heuristic algorithm with polynomial time complexity to solve the optimization problem. We conduct extensive simulations with various parameter settings. Our simulation results demonstrate that our heuristic algorithm gives high quality solutions for the optimization problem.

The remainder of this paper is organized as follows. Section II reviews the related work. Section III formulates the problem and Section IV describes how we obtain the throughput of stations. Our algorithm is presented in Section V and Section VI describes our simulations and presents the performance of our algorithm. And lastly, Section VII concludes this paper.

II. RELATED WORK

A. Unique Features of the IEEE 802.11ax Standard

The first two IEEE 802.11ax drafts, D1.0 and D2.0, were released in 2016 and 2017 [5], respectively. The latest one, D3.0, was released in 2018 [3]. IEEE 802.11ax aims at improving the throughput by a factor of at least 4 [6] as compared to 802.11n/ac in dense scenarios [7]. IEEE 802.11ax has the following features. 1) The use of OFDMA [8] [9] which employs multiple subcarriers that are divided into multiple RUs where RUs are allocated to stations for supporting simultaneous transmission [10] [11]. 2) The use of Downlink/Uplink multi-user multiple-input multiple-output (DL/UL MU MIMO) [12] [13], which improves the throughput by using multiple spatial streams. 3) The use of trigger frame such that AP can coordinate the concurrent transmissions of stations, and that it indicates the number of spatial streams and/or the RU size of each station. 4) The use of random access protocol [14] [15] such that when an AP senses that some stations are going to transmit, but does not know which stations they are, it can assign some RUs for multiple stations to transmit through a random access mechanism.

B. IEEE 802.11ax-Based Dense WiFi Network

IEEE 802.11ax-based dense WiFi network has been attracting researchers' attention recently. Bellalta et al. [1] present some of the network-level functionalities that are required to improve the user experience in dense WiFi scenarios. Deng et al. [2] point out that IEEE 802.11ax will fuel the future intelligent information infrastructure for big data transfer and various mobile applications. Deng et al. [16] discuss the challenges for IEEE 802.11ax in the design of physical layer and medium access control (MAC) sub-layer. Furthermore, they present the expected features on the MAC protocol design to provide better QoS support in the IEEE 802.11ax-based dense WiFi network [17]. Afaqui et al. [18] disclose advanced technological enhancements presented in IEEE 802.11ax to improve the user throughput within a dense WiFi network. All the above results show that IEEE 802.11ax-based dense WiFi network will become popular in the near future.

C. AP Placement and Fault Tolerance

AP placement and fault tolerance have been intensively investigated. Ling et al. [19] jointly solve the two problems of AP placement and channel assignment for providing better network services. Zheng et al. [20] study the AP placement problem aiming to minimize the number of APs being used. In [21], an AP placement problem is formulated, whose objective is to determine the optimal placement of APs. Zhang et al. [22] address the AP placement problem that AP can be equipped with multiple radios to minimize the total cost of all APs. Zhang [23] et al. present an optimization framework of AP placement, whose aim is to maximize the signal coverage. Kiran [24] et al. focus on the optimization of the AP placement to maximize the coverage by optimizing the power allocation. Audhya [25] et al. optimally place the APs in an ultra-dense 5G network to cover a given region. In addition, Zhou et al. [4] study the problem of enhancing the fault tolerance of a WiFi network. They consider the situation that when an AP fails, the stations it serves shall switch to other APs to obtain acceptable services. Liu et al. [26] propose a self-healing scheme to provide a continuous service for users in ultra-dense network. Moreover, Lee et al. [27] propose a resource allocation algorithm to overcome the unforeseen AP failures. These works, however, do not consider the joint design of AP placement and power-channel-RU assignment.

III. Problem Formulation

The symbols used in our model are shown in Table II.

TABLE II
SYMBOLS USED IN OUR MODEL

Symb	Meaning	Symb	Meaning
A	The set of APs	A_f	The set of fault APs
S	The set of stations (STA)	B	The set of channel widths
P	The set of power levels	C	The set of channels
K	Subcarrier number set	R_i	The data rate of STA i
δ_i	The throughput of STA i	$\delta_i^{(H)}$	1 if $\delta_i \geq \rho_H$, 0 otherwise
$I_{i,j}$	The interference range between APs i and j	$\delta_i^{(L)}$	1 if $\rho_H > \delta_i \geq \rho_L$, 0 otherwise
Ω	The set of AP candidate locations	$N(i)$	The set of neighboring APs of AP i
$A(i)$	The set of APs that cover STA i	$S(i)$	The set of STAs that associate with AP i

A. The Network Model

We assume that a set of possible AP locations, denoted by Ω, which is given beforehand. That is, the target region is divided into multiple cells and APs can be placed only at the center of them. Within one cell location, zero, one, or more APs can be placed according to the density of the stations (STAs). Our network consists of three kinds of devices: network controller [28], APs, and STAs. The network controller is responsible for network management and coordination. APs do not need to backoff before transmission under the coordination of the network controller. Denote S and A to be the set of STAs and the set of APs, respectively. Any STA $i \in S$ can only associate with one AP $j \in A$ during a period of time. When a set of APs, A_f ($|A_f| < |A|$), fail, the STAs which are previously associated with the failed APs can re-associate with other APs $\in A \setminus A_f$ to obtain the WiFi service. We adopt both the 2.4 and 5 GHz bands, in which a channel with b MHz ($b \in B = \{20, 40, 80, 160\}$ MHz [3]) bandwidth can be assigned to an AP. In our model, the OFDMA physical layer is adopted. Instead of the whole channel, the RUs are assigned to STAs for simultaneous transmissions. Each AP can only be assigned a channel in a given channel set, denoted by C. Each STA can only be assigned a j-tone RU [3] ($j \in K$, where K is the set of subcarrier numbers). In addition, each AP is assigned a power level within a given power level set, denoted by P. The power level of each STA is the same as that of its AP [21]. The frame exchange procedure is shown in Fig. 1 [17], where TXOP, SIFS, M-BA, and OFDMA-BA represent the transmission opportunity, the short interframe space, the multi-station block ACK, and the OFDMA block ACK, respectively [3]. Under OFDMA, STAs start to transmit Uplink physical layer protocol data unit (PPDU) to its AP only after receiving a trigger frame (TF). The STAs reply an OFDMA-BA frame to the AP after receiving the Downlink PPDU.

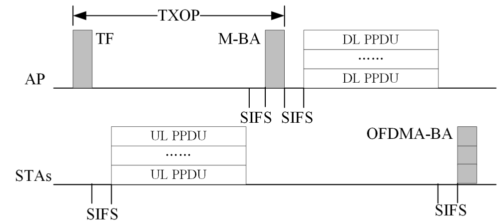


Fig. 1. Frame exchange procedure between an AP and its STAs [17].

B. The Interference Model

Let $l_{i,j}$ denote the link between nodes i and j (here, a node refers to an AP or a STA). To let node i receive frame properly from j over link $l_{i,j}$, the received signal strength (RSS) at node i must be no less than the frame decoding threshold θ_D [22]. In this case, we say that node i is within the transmission range of j and vice versa. In addition, node i is said to be interfered by j (here, nodes i and j are on different links whose channels overlap with each other) if the signal strength received by node i from j is greater than or equal to the interference signal strength threshold θ_I [22]. In this case, we say that node i is within the interference range of j and vice versa. Usually, $\theta_D > \theta_I$. To obtain the communication ranges and the interference ranges of APs, we resort to the following path loss model [29]: $RSS = P_j + G_{TX} - P_{lost} + G_{RX}$, where $P_{lost} = P_{ref} + 10\lg(d^\eta) + \chi$. Here, RSS is the received signal strength at the receiver; d is the distance between the sender and the receiver; P_j is the power level of sender node j; G_{TX} and G_{RX} are the antenna gains of the sender and the receiver respectively; P_{ref} is the path loss at a reference distance (which is usually 1 m); η is the path-loss exponent; and χ is the standard deviation associated with the degree of shadow fading. Thus, $d = \sqrt[\eta]{10^{(P_j + G_{TX} - P_{ref} - \chi + G_{RX} - RSS)/10}}$. Let r_j and γ_j denote the communication range and the interference range of node j, respectively. We have $d = r_j$ if $RSS = \theta_D$ and $d = \gamma_j$ if $RSS = \theta_I$. Next, we introduce the interference model [30]. Let $l_{i,x}$ and $l_{j,y}$ denote the links between AP i and STA x, and AP j and STA y, respectively. Let $d_{i,x}$ and $d_{j,y}$ denote the distance between AP i and STA x, and AP j and STA y, respectively. Let γ_x and γ_y denote the interference range of STA x and STA y, respectively. Fig. 2 depicts the interference ranges of links $l_{i,x}$ (the region enclosed by dotted line) and $l_{j,y}$ (the region enclosed by solid line).

Denote $S(i)$ and $S(j)$ the sets of STAs that are associated with APs i and j, respectively. According to Fig.2, we define the interference range between APs i and j as

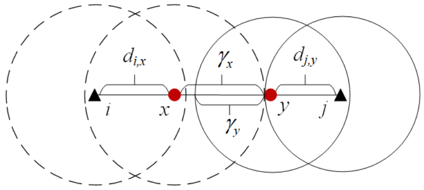


Fig. 2. Interference range between APs i and j [30].

Procedure 1: Obtaining the throughput of STAs

Input : A, S, P, C, etc.
Output: δ_i $(i \in S)$.
Step 1. STAs-APs association.
Step 2. Power adjustment for the APs.
Step 3. Channel assignment and power re-adjustment for the APs.
Step 4. RU assignment for the STAs.
Step 5. Obtaining the data rate of STAs.
Step 6. Calculating the throughput of STAs.

$I_{i,j} = \max_{x \in S(i)}\{d_{i,x}\} + \max\{\gamma_x, \gamma_y\} + \max_{y \in S(j)}\{d_{j,y}\}$. If the distance between APs i and j is less than or equal to $I_{i,j}(i \neq j)$ and their channels are overlapping with each other, then links $l_{i,x}$ and $l_{j,y}$ interfere with each other. That is, they cannot transmit simultaneously [22].

C. The Optimization Problem

Denote δ_i the throughput of STA i. Our problem can be formulated as

$$\min |A|$$
$$s.t. \begin{cases} \text{C1:} & \sum_{j=1}^{|A \setminus A_f|} a_{i,j} = 1, i \in S; \\ \text{C2:} & \sum_{i=1}^{|S|} \delta_i^{(H)} \geq |S| \times \beta\%; \\ \text{C3:} & \sum_{i=1}^{|S|} (\delta_i^{(L)} + \delta_i^{(H)}) = |S|. \end{cases} \tag{1}$$

Here, $a_{i,j}$ is equal to 1 if STA i associates with AP j, and 0 otherwise; $\delta_i^{(H)}$ is equal to 1 if $\delta_i \geq \rho_H$, and 0 otherwise; $\delta_i^{(L)}$ is equal to 1 if $\rho_H > \delta_i \geq \rho_L$, and 0 otherwise. C1 indicates that any STA $i \in S$ can associate with one AP $\in A \setminus A_f$ when $|A_f| = n$ APs fail. C2 ensures that the throughput of at least β % of STAs is greater than or equal to ρ_H. C3 ensures that the throughput of the remaining STAs is greater than or equal to ρ_L. We call C1 the ***fault tolerance requirement***, and C2 and C3 the ***user satisfaction ratio requirement***.

D. The General Framework of Our Solution

To solve (1), δ_i $(i \in S)$ should be obtained first (see Section IV). For a given set S and an AP placement scheme A that satisfies C1 (if A cannot satisfy C1, we add APs to set A until it satisfies C1), we adopt **Procedure 1** to obtain δ_i. After δ_i $(i \in S)$ is obtained, we check whether set A fulfils C2 and C3. If not, we add APs to set A until C2 and C3 can be met.

IV. Throughput of Stations

A. STAs-APs Association

To do the STAs-APs association, we first obtain the set of APs from which their signals can cover STA i, denoted by $A(i)$. We initialize the power level of each AP in P to its maximum level (which will be adjusted later) to cover as many STAs as possible. That is, $P_j = \max_{q \in P}\{p_q\}, j \in A$, where p_q denotes the q-th power level in P. If the distance between STA i and AP j, $d_{i,j}$, is less than or equal to the communication range of AP j, r_j, then the signal emitted from AP j can cover STA i. Thus, $A(i) = \{\text{AP } j | d_{i,j} \leq r_j\}, i \in S, j \in A$. Then, we associate STA i to the AP in $A(i)$ whose signal strength received by STA i is the strongest. After the STAs-APs association, the set of STAs associated with AP j, $S(j)$ $(j \in A)$, can also be obtained.

B. Power Adjustment

As mentioned earlier, the power level of each AP is initialized to the maximum in P. But higher power level leads to larger interference range, so we need to downward adjust the power levels of APs to reduce their interference among each other. Denote $r_j^{(q)}$ the communication range of AP j with power level p_q. We have $r_j^{(1)} < r_j^{(2)} < ... < r_j^{(|P|)}$ under the assumption of $p_1 < p_2 < ... < p_{|P|}$. After the STAs-APs association, we know the maximum distance between AP j and its STAs, $\max_{i \in S(j)}\{d_{i,j}\}$. Then the power level of AP j can be adjusted as follows:

$$P_j = \begin{cases} p_q, & \text{if } \max_{i \in S(j)}\{d_{i,j}\} \in (r_j^{(q-1)}, r_j^{(q)}], q \in [2, |P|]; \\ p_1, & \text{if } \max_{i \in S(j)}\{d_{i,j}\} \leq r_j^{(1)}. \end{cases} \tag{2}$$

With P_j $(j \in A)$ in hands, we can further obtain the interference range between APs i and j, $I_{i,j}(i \neq j)$, which can help to assign channels to the APs.

C. Channel Assignment and Power Re-adjustment

We use both the 2.4 and 5 GHz [31] bands for transmissions. The channels used in our model are shown in Fig. 3. Thus,

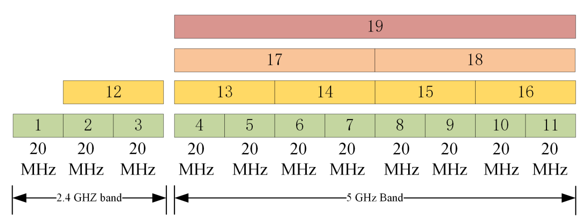


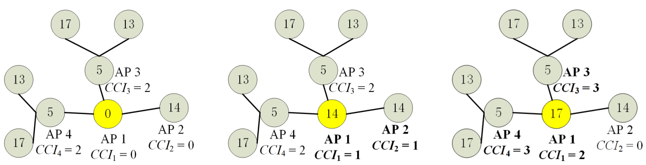
Fig. 3. The channels used in our model.

the channel set $C = \{1, 2, ..., 19\}$. From Fig. 3 we observe that channel 12 overlaps with channels 2 and 3. Hence, we define two overlapping channel sets (OCSs) in the 2.4 GHz band, denoted by $\Gamma_1 = \{2, 12\}$ and $\Gamma_2 = \{3, 12\}$, respectively. Similarly, we define eight OCSs in the 5 GHz band, denoted by $\Gamma_3 = \{4, 13, 17, 19\}$, $\Gamma_4 = \{5, 13, 17, 19\}$, ..., $\Gamma_{10} = \{11, 16, 18, 19\}$. We call channels $1 \sim 11$ *primary channels*.

Denote the set of neighboring APs of AP i by $N(i)$, which is defined as $N(i) = \{\text{AP } j | D_{i,j} \leq I_{i,j}\}, i, j \in A, i \neq j$, where $D_{i,j}$ denotes the distance between APs i and j. For any AP i, if AP $j \in N(i)$ and the non-overlapping channels are enough, then we assign a channel which does not overlap with AP j's channel to AP i. When the non-overlapping channels are not enough, the channel assigned to AP i may overlap with AP

j's channel. In this case, we try to reduce the total interference degree between AP i and its neighbors (see next paragraph). Obviously, if AP $j \notin N(i)$, then the channel assigned to AP i can be the same as that being assigned to AP j.

We introduce a channel conflict indicator (CCI) to measure the interference degree of APs. Let CCI_i denote the interference degree of AP i, which is defined as: the number of AP i's neighbors whose channels belong to the same OCS as the channel of AP i. A CCI graph is presented as shown in Fig. 4 to help us assign the channels. In Fig. 4, the circle represents the AP whose channel number is presented at the center. The two APs connected by an edge are neighbors to each other. For example, in Fig. 4(a), we want to assign a channel to AP 1 whose channel number is initialized as 0. Suppose that no other channels can be used. Thus, we can choose one only from {5, 13, 14, 17} for AP 1. Notice that 5, 13, 17 $\in \mathit{\Gamma}_4$, we assign 14 to AP 1 (see Fig. 4(b)). The reason is that any one from {5, 13, 17}, such as 17, assigning to AP 1 increases the CCI values of AP 1 and its neighbors significantly (see Fig. 4(c)), which leads to a higher degree of interference.



(a) Initial CCI value. (b) AP 1 $\leftarrow$ 14. (c) AP 1 $\leftarrow$ 17.

Fig. 4. CCI graph.

The channel assignment algorithm is shown in Algorithm 1. At the beginning, the channel numbers and CCI values of all APs are initialized to 0. First, we generate a channel assignment queue Q_a by sorting the APs in descending order according to the number of STAs served by each AP. Then we assign channels to APs one by one according to the AP order in Q_a (we construct corresponding CCI graph at the same time). That is, the APs with more STAs are given higher priority to be assigned to the channels. In Algorithm 1, $c_{i,j}$ is equal to 1 if channel j $(j \in C)$ is assigned to AP i $(i \in A)$, and 0 otherwise. Lines 8 $\sim$ 9 mean that we assign the first channel (i.e., the primary channel) in set C^* to AP i when the non-overlapping channels are enough (i.e., $C^* \neq \varnothing$). Lines 10 $\sim$ 13 mean that when the non-overlapping channels are not enough, AP i is always assigned the channel with the minimum increase of CCI values of AP i and its neighbors. Lines 14 $\sim$ 16 update the channels that have been assigned to the APs to channels with wider bandwidth.

After the channel assignment, we re-adjust the power level of APs to increase the RSS of STAs (i.e., to increase their data rate). For each AP i whose power level p_q is lower than $p_{|P|}$, we adjust its power level from p_q to p_{q+1} $(q \in [1, |P|-1])$, and then judge whether the signals emitted from AP i with power level p_{q+1} interfere with other basic service sets. If so, then we restore the power level to its original value (i.e., p_q) and stop; otherwise, we continue to adjust its power level from

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Algorithm 1: Channel assignment

Input : A, C, $\mathit{\Gamma}_u$ $(u = 1, 2, ..., 10)$, $N(i)$ $(i \in A)$.
Output: $c_{i,j}$, $(i \in A, j \in C)$.

1 $j \leftarrow 0$; $c_{i,j} \leftarrow 1$; $CCI_i \leftarrow 0$ $(i \in A)$;
2 Generate queue Q_a;
3 **for** each AP $i \in Q_a$ **do**
4     $C^* \leftarrow C$;
5   **for** each AP $i' \in N(i)$ **do**
6   **if** $j' \in \mathit{\Gamma}_u$ $(u = 1, 2, ..., 10) \wedge c_{i',j'} = 1$ **then**
7             $C^* \leftarrow C^* \setminus \mathit{\Gamma}_u$;
8   **if** $C^* \neq \varnothing$ **then**
9         $j \leftarrow C^*(1)$; $c_{i,j} \leftarrow 1$;
10   **else**
11   Select a channel j in set C with the minimum increase of CCI values of AP i and its neighbors;
12         $c_{i,j} \leftarrow 1$;
13   Update CCI values of AP i and its neighbors;
14 **for** each AP $i \in Q_a$ **do**
15   **if** the bandwidth of channel $j'(j' \in C)$ is wider than that of channel j $\wedge$ assigning channel j' to AP i does not increase CCI values of AP i and its neighbors **then**
16         $j \leftarrow j'$; $c_{i,j} \leftarrow 1$;

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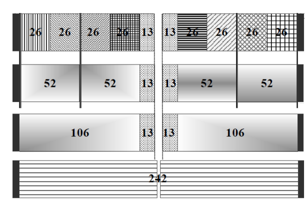
p_{q+1} to p_{q+2} until the power level is equal to $p_{|P|}$.

D. RU Assignment

There are seven types of RUs defined in IEEE 802.11ax [3], that is, the subcarrier (i.e., tone) number set $K = \{26, 52, 106, 242, 484, 996, 2 \times 996\}$. The maximum number of k-tone RUs $(k \in K)$ for each channel width are shown in Fig. 5(a), where $b_1 = 20$, $b_2 = 40$, $b_3 = 80$, and $b_4 = 160$ (MHz) [3]. It implies that up to 9 STAs in 20 MHz, 18 STAs in 40 MHz, 37 STAs in 80 MHz, and 74 STAs in 160 MHz channel width are supported in an OFDMA transmission. Fig. 5(b) shows the RU locations in a 20 MHz PPDU [3]. The RU locations in a 40, 80, or 160 MHz PPDU can be found in [3]. As shown in Fig. 5, the maximum number of k-tone RUs

RU Type	b_1	b_2	b_3	b_4
26-tone	9	18	37	74
52-tone	4	8	16	32
...	...	...	...	...
2×996-tone	N/A	N/A	N/A	1

(a) Maximum number of RUs.



(b) RU locations (20 MHz).

Fig. 5. Maximum number of RUs and their locations [3].

$(k \in K)$ is determined by the channel width. Hence, when assigning RUs to AP i's STAs, we consider the number of STAs served by AP i, $|S(i)|$, as well as the channel width of AP i. We mainly focus on the following two aspects: 1) how to utilize the bandwidth of AP i's channel as much as possible; 2) how to balance the data rate of STAs in $S(i)$ as much as possible. **For aspect one**, we define m_b RU sets, $RU_{b,m}$ $(b \in B, m = 1, 2, ..., m_b$, where m_b is the maximum number of 26-tone RUs in b MHz channel width), for RU assignment and guarantee that the total bandwidth of the RUs in set $RU_{b,m}$

is as close to b as possible. For example, according to Fig. 5(b), we define $RU_{20,1}$ = {242} for 1 STA, $RU_{20,2}$ = {106, 106} for 2 STAs, $RU_{20,3}$ = {26, 106, 106} for 3 STAs, ..., $RU_{20,9}$ = {26, 26, 26, 26, 26, 26, 26, 26, 26} for 9 STAs. **For aspect two**, we assign larger-size RUs to STAs that are farther away from AP i and smaller-size RUs to STAs that are nearer to AP i. For any AP i with channel width b, we adopt the following steps to assign RUs to its STAs.

Step 1. We divide $|S(i)|$ by m_b to get the quotient of $\lfloor S(i)/m_b \rfloor$ with the remainder of rem.

Step 2. We divide AP i' $|S(i)|$ STAs into $\lfloor S(i)/m_b \rfloor + 1$ groups. The x-th group ($x = 1, 2, ..., \lfloor S(i)/m_b \rfloor$) contains m_b STAs and the ($\lfloor S(i)/m_b \rfloor + 1$)-th group contains rem STAs.

Step 3. We assign the RUs in set RU_{b,m_b} to the STAs in the x-th group ($x = 1, 2, ..., \lfloor S(i)/m_b \rfloor$), and assign the RUs in set $RU_{b,rem}$ to the STAs in the ($\lfloor S(i)/m_b \rfloor + 1$)-th group.

These $\lfloor S(i)/m_b \rfloor + 1$ groups of AP i's STAs exchange frames with AP i in turn.

E. Data Rate of STAs

We obtain the data rate of STAs according to the RSS and the RU of STAs. From [3], we can get the correspondence between the receiver minimum input level sensitivity and data rate, that is, ($MS_{b,1}, \sigma_{k,1}$), ($MS_{b,2}, \sigma_{k,2}$), ..., ($MS_{b,X}, \sigma_{k,X}$), where $MS_{b,x}$ ($b \in B, x = 1, 2, ..., X$) denotes the x-th minimum sensitivity (MS) in b MHz channel width; $\sigma_{k,x}$ denotes the x-th data rate for k-tone RU ($k \in K$). Here, $MS_{b,1} < MS_{b,2} < ... < MS_{b,X}$, and $\sigma_{k,1} < \sigma_{k,2} < ... < \sigma_{k,X}$. The data rate of an UL (or DL) from STA i to AP j (or from AP j to STA i) depends on the RSS at AP j (or STA i) [22]. Denote RSS_i the signal strength received by AP j from STA i, and R_i the data rate of STA i for UL data traffic. Thus, we have $R_i = \sigma_{k,q}$ if $MS_{b,q} \leq RSS_i < MS_{b,q+1}, q \in [1, X-1]$; and $R_i = \sigma_{k,X}$ if $MS_{b,X} \leq RSS_i$. Since we assume that the power level of STA i's AP is the same as that of STA i [21], the data rate of STA i for DL data traffic is the same as R_i.

F. Throughput of STAs

For any AP i with channel width b, there are $|S(i)|$ STAs associated with it. Thus, it needs

$$M = \begin{cases} |S(i)|/m_b, & \text{if } |S(i)| = zm_b, z \in Z^+; \\ \lfloor |S(i)|/m_b \rfloor + 1, & \text{otherwise}, \end{cases} \tag{3}$$

frame exchanges (including DL and UL traffics) to complete one communication round (i.e., each STA in $S(i)$ completes one UL transmission and one DL reception).

Let t_{TF}, t_{SIFS}, t_{UL_PPDU}, t_{M_BA}, t_{DL_PPDU}, and t_{OFDMA_BA} to denote the duration of TF, SIFS, UL PPDU, M-BA, DL PPDU, and OFDMA-BA, respectively. Denote the duration of an UL transmission by T_{UL}, the duration of a DL reception by T_{DL}. According to Fig. 1, we have $T_{UL} = t_{TF} + 2t_{SIFS} + t_{UL\_PPDU} + t_{M\_BA}$ and $T_{DL} = 2t_{SIFS} + t_{DL\_PPDU} + t_{OFDMA\_BA}$. Denote the duration of one communication round by T, then $T = (T_{UL} + T_{DL})M$. Thus, the throughput of STA i associated with AP j is

$$\delta_i = \frac{R_i(t_{UL\_PPDU} + t_{DL\_PPDU})}{T(CCI_j + 1)}, \tag{4}$$

where $CCI_j + 1$ means that AP j and its CCI_j neighbors interfere with each other. That is, they must be active in turn.

V. Algorithm design and Analysis

Since the sub-problems arise from our problem (such as the optimal AP placement etc.) are NP-hard [22], our problem is therefore NP-hard as well. Thus, we design a polynomial time heuristic algorithm to providing a solution for it.

A. The Heuristic Algorithm

Our heuristic algorithm consists of four stages (see below for technical details in each stage). The key operation in each stage is to check whether a given solution A is feasible. Thus, we first design Algorithm 2 for the feasibility test.

Algorithm 2: Feasibility test for solution A

Input : A, S, Ω, P, C, n, etc.
Output: an indicator I (if A is feasible, then $I = TRUE$; otherwise $I = FALSE$).

$I \leftarrow TRUE$; $A^* \leftarrow A$;
if C1 can be met **then**
  for each $A_f \subset A$ **do**
    $A \leftarrow A^*$;
    $A \leftarrow A \setminus A_f$;
    Call **Procedure 1** to obtain the throughput of STAs;
    if C2 and C3 cannot be met **then**
      $I \leftarrow FALSE$;
      Quit the loop;
else
  $I \leftarrow FALSE$;
Return I;

In stage one, we use the Greedy algorithm as shown in Algorithm 3 to iteratively place AP at the location around which the density of uncovered STAs is the highest until all STAs are covered and C1 $\sim$ C3 in (1) can be met to get an initial solution A_1.

Algorithm 3: Stage one-constructing an initial set of APs

Input : S, Ω, P, C, n, etc.
Output: A_1.

$A_1 \leftarrow \varnothing$; $i \leftarrow 0$;
repeat
  $i \leftarrow i + 1$;
  Place AP i at the location around which the density of uncovered STAs is the highest;
  $A_1 \leftarrow A_1 \cup$ {AP i};
  Call **Algorithm 2** to test the feasibility of A_1;
  if the output of algorithm 2 is $FALSE$ **then**
    Return to the **repeat** statement;
until the output of algorithm 2 is $TRUE$;
Return A_1;

In stage two, we try to iteratively remove redundant APs in A_1 one by one in a predefined order. That is, in each iteration, we generate an AP queue Q_b by sorting the APs in ascending order according to the number of STAs served by each AP. Then we try to remove the first AP in Q_b. If it cannot be removed (i.e., once it is removed, C1, C2 or C3 can not be met), then we keep it and try to remove the next one in Q_b;

otherwise, we remove it and start the next iteration. The reason why we always try to remove the AP at the head of Q_b first in each iteration is that the AP with the least number of STAs is more likely to be removed. The operations of stage two are shown in Algorithm 4.

Algorithm 4: Stage two-removing the redundant APs

Input : A_1.
Output: A_2.

$A_2 \leftarrow A_1$;
repeat
  Generate queue Q_b;
  for each AP $i \in Q_b$ **do**
    $A_2 \leftarrow A_2 \setminus \{\text{AP } i\}$;
    Call **Algorithm 2** to test the feasibility of A_2;
    if the output of algorithm 2 is $FALSE$ **then**
      $A_2 \leftarrow A_2 \cup \{\text{AP } i\}$;
    else
      Return to the **repeat** statement;
until $i = |Q_b|$;
Return A_2;

In stage three, we iteratively replace two nearby APs in set A_2 by one. There are two main tasks in each iteration: 1) finding the pair of APs with the shortest distance; and 2) trying to replace the pair of APs by a new one with a feasible location in Ω. The main steps in each iteration are as follows:

Step 1. We generate $\binom{|A_2|}{2}$ pairs of APs and calculate the distance of each pair of APs.

Step 2. We generate a replacement queue $Q_c[i] = \{\text{AP } i_1, \text{AP } i_2\}$, where APs i_1 and i_2 represent the two APs of the i-th pair of APs ($i = 1, 2, ..., \binom{|A_2|}{2}$), by sorting the $\binom{|A_2|}{2}$ pairs of APs in ascending order according to the distance of each pair of APs.

Step 3. We iteratively try to replace the pair of APs at the head of Q_c by one until all pairs of APs have been tried.

The operations of stage three are shown in Algorithm 5.

Algorithm 5: Stage three-replacing two nearby APs by one

Input : A_2.
Output: A_3.

$A_3 \leftarrow A_2$;
repeat
  Generate queue Q_c;
  for each $Q_c[i] \in Q_c$ **do**
    $A_3 \leftarrow A_3 \setminus Q_c[i]$;
    for each location $g \in \Omega$ **do**
      Place a new AP at location g;
      $A_3 \leftarrow A_3 \cup \{\text{the new AP}\}$;
      Call **Algorithm 2** to test the feasibility of A_3;
      if the output of algorithm 2 is $FALSE$ **then**
        $A_3 \leftarrow A_3 \cup Q_c[i]$;
        $A_3 \leftarrow A_3 \setminus \{\text{the new AP}\}$;
      else
        Return to the **repeat** statement;
until $i = |Q_c|$;
Return A_3;

In stage four, we continue to try to reduce the number of APs by iteratively replacing three nearby APs in set A_3 by two. The main steps in each iteration are as follows:

Step 1. We generate $\binom{|A_3|}{3}$ groups of APs with each containing 3 APs.

Step 2. We calculate the distance between APs i_1 and i_2, D_{i_1,i_2}; APs i_2 and i_3, D_{i_2,i_3}; and APs i_1 and i_3, D_{i_1,i_3}, respectively, where APs i_1, i_2, and i_3 denote the three APs of the i-th group ($i = 1, 2, ..., \binom{|A_3|}{3}$).

Step 3. We calculate the distance between APs i_1, i_2, and i_3, G_i, which is defined by $G_i = D_{i_1,i_2} + D_{i_2,i_3} + D_{i_1,i_3}$.

Step 4. We generate a replacement queue $Q_d[i] = \{\text{AP } i_1, \text{AP } i_2, \text{AP } i_3\}$ by sorting the $\binom{|A_3|}{3}$ groups of APs in ascending order according to the values of G_i ($i = 1, 2, ..., \binom{|A_3|}{3}$).

Step 5. We iteratively try to replace the group of APs at the head of Q_d by two until no group of APs can be replaced.

The operations of stage four are shown in Algorithm 6.

Algorithm 6: Stage four-replacing three nearby APs by two

Input : A_3.
Output: A_4.

$A_4 \leftarrow A_3$;
repeat
  Generate queue Q_d;
  for each $Q_d[i] \in Q_d$ **do**
    $A_4 \leftarrow A_4 \setminus Q_d[i]$;
    for each pair of locations $g_1, g_2 \in \Omega$ **do**
      Place two new APs at locations g_1 and g_2;
      $A_4 \leftarrow A_4 \cup \{\text{the two APs}\}$;
      Call **Algorithm 2** to test the feasibility of A_4;
      if the output of algorithm 2 is $FALSE$ **then**
        $A_4 \leftarrow A_4 \cup Q_d[i]$;
        $A_4 \leftarrow A_4 \setminus \{\text{the two APs}\}$;
      else
        Return to the **repeat** statement;
until $i = |Q_d|$;
Return A_4;

Actually, we may continue to iteratively replace x nearby APs by $x - 1$ ($x = 4, 5, ...$), but there are two reasons that prevent us from doing that. The first one is about the computation time and time complexity for the next solution. The second one is about how much improvement we can obtain as compared to our last algorithm (see Section VI).

B. Time Complexity Analysis

The time complexity of our algorithm is determined by Algorithm 6, which calls Algorithm 2 to test the feasibility of the current AP placement scheme. In addition, Algorithm 2 calls Procedure 1 to obtain the throughput of STAs. Therefore, the time complexity of our algorithm is the product of the time complexity of Algorithm 6, Algorithm 2, and Procedure 1. Suppose that the input of Algorithm 6 is A. In the worst case, we have to do $|Q_d| = \binom{|A|}{3}$ attempts to replace three APs by two. In each replacement attempt, we should search $(\binom{|\Omega|}{2} + |\Omega|)$ times to find a feasible pair of locations for two new APs (if two new APs do not overlap, there are $\binom{|\Omega|}{2}$ location combinations; if they overlap, there are $|\Omega|$ candidate locations). Thus, the time complexity of Algorithm 6 is $O(\binom{|A|}{3}(\binom{|\Omega|}{2} + |\Omega|)) = O(|A|^3|\Omega|^2)$. In Algorithm 2, for the network that contains $|A|$ APs, when n APs fail, there are $\binom{|A|}{n}$ AP fault combinations need to be checked for testing the feasibility of A. That is, the time complexity of Algorithm 2 is $O(\binom{|A|}{n}) = O(|A|^n)$. The time complexity of Procedure 1 is equal to the sum of the time complexity of its six steps, namely, $O(|S| + |A| + (|A|^2 + |A|) + |S| + |S| + |S|)$, where $(|A|^2 + |A|)$ is the time complexity of channel assignment and power re-adjustment. Thus, the time complexity of our algorithm is $O(|A|^{3+n}|\Omega|^2(4|S| + |A|^2))$.

VI. Performance Evaluation

A. Simulation Settings

Since NS3 does not currently support some of the major functions (such as MU-OFDMA) of IEEE 802.11ax [32], we develop a simulator using MATLAB [33]. We set $C = \{1, 2, ..., 19\}$ as shown in Fig. 3 and $P = \{14, 15, 16, 17\}$ dBm. For the path loss model [29], we set $G_{TX} = 4$ dBi, $G_{RX} = 4$ dBi, $P_{ref} = 30$ dB, $\eta = 4$, and $\chi = 5$ dB. The frame decoding threshold and the interference signal strength threshold are set as: $\theta_D = -68$ dBm and $\theta_I = -77$ dBm. The RUs and the correspondence between the receiver minimum input level sensitivity and the data rate can be found in [3], which are applied to assign the RUs to STAs and obtain the data rate of STAs, i.e., R_i $(i \in S)$. For the throughput of STAs under the OFDMA mechanism (see Fig. 1), we set $L_{TF} = 68$ B, $L_{M\_BA} = 118$ B, and $L_{OFDMA\_BA} = 32$ B, where L_{TF}, L_{M_BA}, and L_{OFDMA_BA} represent the size of TF, M-BA, and OFDMA-BA, respectively. When transmitting control frames (i.e., TF, M-BA, and OFDMA-BA), the data rate of APs/STAs is set as 7.5 Mbps [3]. The SIFS duration, t_{SIFS}, is set as 10×10^{-6} s when operating in the 2.4 GHz band, and 16×10^{-6} s when operating in the 5 GHz band [34]. We set the TXOP as 3×10^{-3} s [35], which yields $t_{UL\_PPDU} = 3 \times 10^{-3} - 2t_{SIFS} - t_{TF} - t_{M\_BA}$. We suppose that the ratio of UL to DL duration is 1/2, which yields $t_{DL\_PPDU} = 2t_{UL\_PPDU}$. We assume that the lowest throughput that users can accept is 0.5 Mbps, that is, we set $\rho_L = 0.5$ Mbps. The following results are from an average of 30 simulation runs. In each run, the locations of STAs are generated randomly.

B. Effectiveness Evaluation

In this part, we set the target region as 50×50 m^2, which is divided into 25 10×10 m^2 cells. Thus, $|\Omega| = 25$.

Considering that the throughput model in (4) plays a key role in our algorithm, we verify it first via the comparison of the simulation and the analytical results. Setting $|S| = 500$, $\beta = 90$, $n = 0$, and $\rho_H = 1$ Mbps, respectively, we obtain Fig. 6. The upper part of the figure compares the simulation results with the analytical ones derived directly from (4), while the lower part shows the errors. We can see that the simulation and analytical results match very well since the errors are less than 2%. Therefore, the throughput model in (4) is verified.

We then evaluate the effectiveness of the four algorithms by comparing it with the exhaustive search (ES) method. The four algorithms are as follows: one-stage: stage 1; two-stage: stages $1 \sim 2$; three-stage: stages $1 \sim 3$; and four-stage: stages

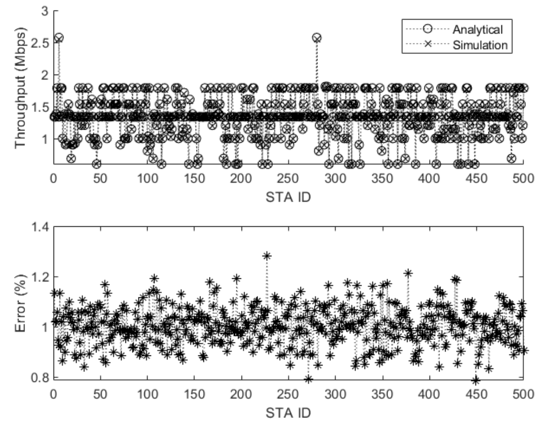


Fig. 6. Comparison of simulation and analytical results ($|S| = 500$).

$1 \sim 4$. Setting $|S| = 100, 200, 300, 400, 500$, $\beta = 90$, $n = 0$, and $\rho_H = 1$ Mbps, respectively, we obtain Tables III and IV, in which the data from row 2 to 6 represent the average number of APs and the average execution time of 30 runs, respectively. Table III shows that the number of APs obtained from the four-stage is equal to that obtained from the ES, namely, our four-stage algorithm performs as good as the ES under the small-scaled case. Table IV shows that the four-stage saves $41 \sim 82$ % execution time as compared to ES. In short, our algorithm is very effective in providing a solution.

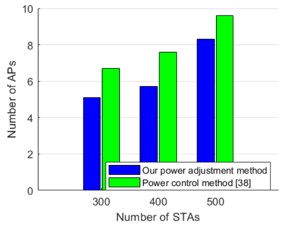
TABLE III
Comparison of the number of APs

$\|S\|$	**100**	**200**	**300**	**400**	**500**
ES method	2.00	3.00	3.00	3.20	4.00
Four-stage	2.00	3.00	3.00	3.20	4.00
Three-stage	2.10	3.00	3.73	3.83	5.07
Two-stage	2.57	3.07	4.13	5.40	7.53
One-stage	2.97	3.27	4.70	6.17	8.80

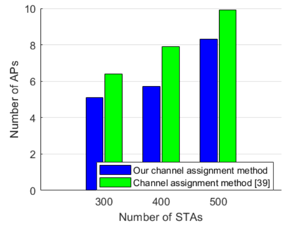
TABLE IV
Comparison of the execution time (second)

$\|S\|$	**100**	**200**	**300**	**400**	**500**
ES method	0.17	1.02	3.46	24.45	64.83
Four-stage	0.10	0.35	1.24	4.42	18.64
Three-stage	0.09	0.18	0.81	2.48	8.34
Two-stage	0.06	0.10	0.28	0.69	1.75
One-stage	0.04	0.06	0.14	0.28	0.59

Next, we compare our algorithm for individual subproblems against solutions from the literature. We replace our power adjustment method and channel assignment method in the four-stage algorithm by the method presented in [36] and [37], respectively, and compare them with the four-stage algorithm. Setting $|S| = 300, 400, 500, \beta = 90, n = 1, \rho_H = 1$ Mbps, respectively, we obtain Fig. 7, which illustrates that our power adjustment method and channel assignment method are better than existing ones.



(a) Power assignment methods.

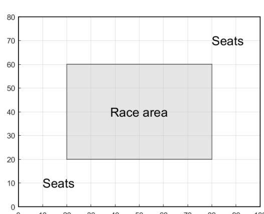


(b) Channel assignment methods.

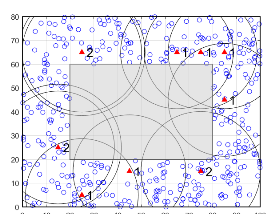
Fig. 7. Comparison of power and channel assignment methods.

C. Performance Evaluation

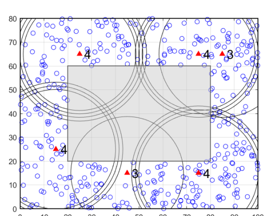
Because our problem is quite different from the others (see Table I), it is difficult to find suitable algorithms to compare with our algorithm. Thus, we evaluate the performance of our algorithm by comparing it with the Greedy algorithm and the Random methods under the larger area where we cannot find the optimal solution through an exhaustive search. We consider the stadium whose layout is shown in Fig. 8(a). The shaded area is the race area and the blank area has many seats for audience. The target region is the blank area which is divided into $(100 \times 80 - 60 \times 40)/(10 \times 10) = 56$ 10×10 m^2 cells. To illustrate the advantages of our solution intuitively, an example is given in Fig. 8(b) $\sim$ (d), in which the red triangles, the blue circles, and the black circles represent the AP locations, the users, and the communication ranges of APs, respectively. The number next to the triangle is the number of APs being placed at that location. We observe that some locations are placed more than one AP. This is because some areas have denser users than the others. In addition, we allow APs to automatically adjust their power, so the communication ranges of them may be different from each other. Fig. 8 shows our solution is much more efficient than that of the Greedy algorithm and the Random method.



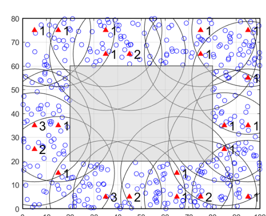
(a) Layout of the stadium.



(b) Our solution (12 APs).



(c) Greedy solution (22 APs).



(d) Random solution (30 APs).

Fig. 8. An example ($|S| = 400, \beta = 90, n = 1, \rho_H = 1$ Mbps).

TABLE V

COST SAVINGS COMPARED WITH GREEDY AND RANDOM (%)

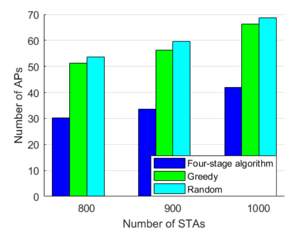
Parameter	$\|S\|$			ρ_H		
Value	800	900	1000	1	1.25	1.5
Compared with Greedy	41.0	40.4	36.6	41.0	32.1	33.2
Compared with Random	43.6	43.8	38.9	43.6	41.8	55.1

TABLE VI

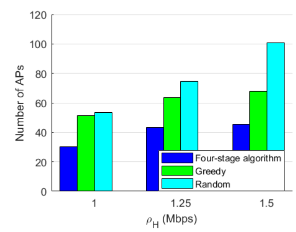
EXECUTION TIME COMPARED WITH GREEDY AND RANDOM (SECOND)

$\|S\|$	**800**	**900**	**1000**
Four-stage	212209.94	433144.79	595376.01
Greedy	147.99	235.86	380.74
Random	198.96	400.35	628.13

Next, we perform the performance comparison with various parameter settings. Setting $|S| = 800, 900, 1000$, $\beta = 90$, $n = 1$, and $\rho_H = 1$ Mbps, respectively, we obtain Fig. 9(a). From Fig. 9(a) we can see that the number of APs obtained from our algorithm is much smaller than that obtained from the Greedy algorithm and the Random method. Setting $\rho_H = 1, 1.25, 1.5$ (Mbps), $|S| = 800$, $\beta = 90$, and $n = 1$, respectively, we obtain Fig. 9(b), which shows that our algorithm produces the best result. Suppose that it costs 1 unit to deploy 1 AP, the cost



(a) $\beta = 90, n = 1, \rho_H = 1$.



(b) $|S| = 800, \beta = 90, n = 1$.

Fig. 9. Performance comparison.

savings of our algorithm are shown in Table V. We can see that our algorithm can save $32 \sim 55$ % in terms of deployment cost. In addition, we compare the execution time of the three methods in Table VI. The table shows that our algorithm has much higher time overhead than Greedy and random methods. The reason is that our algorithm gradually reduces the number of APs through four stages. We leave it as a future work to use parallel computing technique to reduce the execution time.

VII. CONCLUSION

In this paper, we deployed an IEEE 802.11ax-based dense WiFi network under the joint design of AP placement and power-channel-RU assignment. We formulated it as an optimization problem. We then analyzed the throughput of STAs under the mechanism of OFDMA and a widely used interference model and designed a heuristic algorithm with polynomial time complexity to solve the optimization problem which is NP-hard. Simulation results show that our algorithm is efficient and effective in reducing the number of APs.

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