

# Cooperative Stochastic Differential Game in P2P Content Distribution Networks

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# Outline

- Introduction
- Related work
- Analysis of Tit-for-Tat Strategy
- Basic Elements of Game Theory
- Incentive Framework
- Future Work



# Introduction

- Who generates most traffic?

Peer-to-Peer

- Who is the dominating P2P protocol?

BitTorrent

- How to distribute content in a non-cooperative environment?

Swarming

- How to solve the free-riding problem?

... ..(opinions differ)

# Introduction (cont.)

Example: BitTorrent TFT Strategy

➤ Tit-for-Tat (from game theory)

Is it really effective?

**Yes?**

- ✓ Is proved in the repeated prisoner's dilemma.
- ✓ Improves download rate by increases upload rate

# Introduction (cont.)

Example: BitTorrent TFT Strategy

➤ Tit-for-Tat (from game theory)

Is it really effective?

**No!**

- ✓ High variability in download rate
- ✓ Unfairness in terms of ratio of upload & download  
Bandwidth allocation is not Pareto-optimal
- ✓ No reason to contribute once peers have satisfied their immediate demands.

# Introduction (cont.)

- Two Questions
  - Does another strategy exist which outperforms BitTorrent's TFT strategy?
  - Does a strategy exist which ensures fairness between peers although they behave selfishly?
- How to model P2P network?  
strategic, rational, cheat, maximize own payoff, incentive ... ..
  - Game Theory is a proper tool  
But ...

# Introduction (cont.)

- **Our work**
  - Analyze the root issue of TFT strategy
  - Define the basic elements of game theory from P2P content distribution perspective
  - Incentive framework based on cooperative stochastic differential game
    - Payoff distribution procedure
    - Subgame consistency
    - Dynamic Shapley value
    - Equilibrating transitory compensation
    - Follow the original optimality principle and cooperative state trajectory path

# Related Work

- Micro-payment

- relies on centralized server
- Use virtual currency

Short-term  
Incentive

- Game Theory

- Mechanism Design  
J. Shneidman (IPTPS'03)
- Cournot Game  
Richard T.B. Ma (Sigmetrics'04)  
Simon G. M. Koo (Telecommun Syst'07)
- Others  
F. Wu (ACM STOC'07)  
W. Sabrina Lin (ICASSP'08)  
Bridge Q. Zhao (Infocom'09)

Fail to see  
**whole**  
for the  
**part**

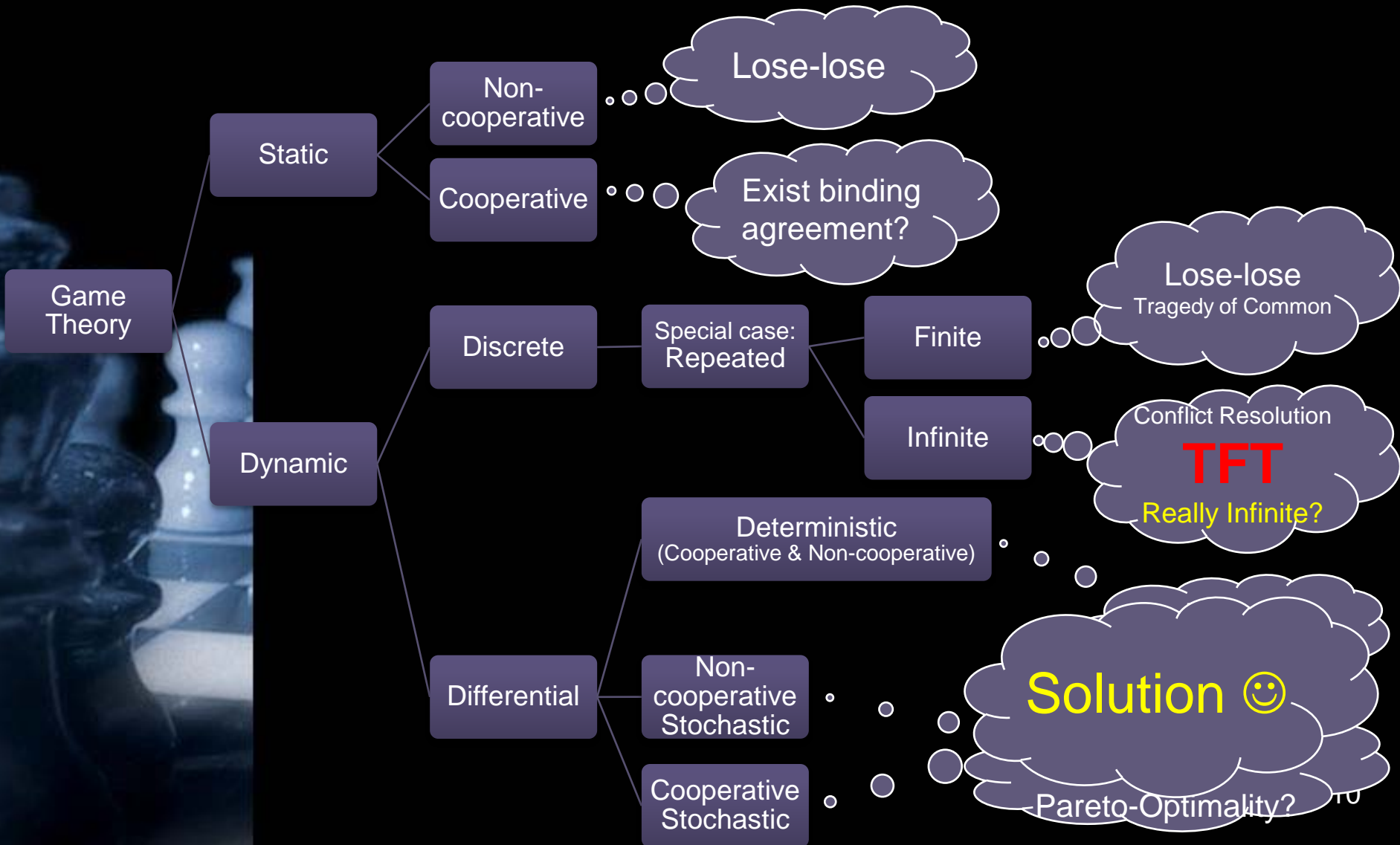
Pareto-optimality!



# Analysis of TFT Strategy

- Non-cooperative environment will be far away from Pareto-optimality  
“solitary, poor, nasty, brutish & shot.”  
-- Tomas Hobbes, <Leviathan>, 1651
- Difficulty of cooperation is rooted from free-riding problem, if do further study, that is:  
Temporary profit of each peer exists conflicts

# Analysis of TFT Strategy (cont.)



# Basic Elements of Game Theory

- Player – peer
- Action – bandwidth peer wants to upload
- Information – peer type, strategy, payoff, etc.
- Strategy – rule or plan, not action only
- Payoff – download bandwidth peer gets
- Rationality – maximizing peer's own payoff
- Objective – optimizes peer's payoff function by strategy or choosing action
- Order of play – selects time point to take action
- Outcome – related terminal values, e.g. reputation
- Equilibrium – combination of optimal strategies

# Incentive Framework

- Stochastic Environment:

Time Consistency → Subgame Consistency

Definition: A cooperative solution is subgame-consistent if an extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behavior would remain optimal.

- How to realize subgame consistency?

Equilibrating transitory compensation  $B_i(s)$

Explanation: player  $i$  receives at time  $s$  given the state is the sum of the following three items:

# Incentive Framework (cont.)

- Equilibrating transitory compensation  $B_i(s)$  is the sum of:
  - Player  $i$ 's agreed upon marginal share of total expected cooperative profit,
  - Player  $i$ 's agreed upon marginal share of his own expected non-cooperative profit plus the instantaneous effect on his non-cooperative expected payoff when the change in the state variable  $x_t^*$  follows the cooperative trajectory instead of the non-cooperative path, and
  - Player  $i$ 's agreed upon marginal share of Player  $j$ 's non-cooperative profit plus the instantaneous effect on Player  $j$ 's non-cooperative payoff when the change in the state variable  $x$  follows the optimal trajectory instead of the non-cooperative path.

# Incentive Framework (cont.)

- Payoff distribution procedure restricted by stochastic differential dynamic system

$$E_{t_0} \left\{ \int_{t_0}^T g^i[s, x_i(s), u_i(s)] \exp\left[-\int_{t_0}^s r(y) dy\right] ds + \exp\left[-\int_{t_0}^s r(y) dy\right] q^i(x_i(T)) \right\}$$

expectation

control/path

instantaneous  
payoff

time

state variable

Discount factor:

$$\exp\left[-\int_{t_0}^s r(y) dy\right]$$

Equilibrating transitory compensation:  $B_i(s)$  is  $g^i[s, x_i(s), u_i(s)]$

Optimal terminal value:  $q^i(x_i(T))$

# Incentive Framework (cont.)

- Dynamic Shapley value  
Condition: At time  $\tau$ , peer  $i$ 's share of profits be:

$$v^{(\tau)i}(\tau, x_N^{\tau*}) = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!} [W^{(\tau)K}(\tau, x_K^{\tau*}) - W^{(\tau)K \setminus i}(\tau, x_{K \setminus i}^{\tau*})]$$

$$W^{(t_0)K}(T, x_K) = \sum_{j \in K} \exp[-\int_{t_0}^T r(y) dy] q^j(x_j)$$

# Incentive Framework (cont.)

- We can prove and get following Theorem: A payment to peer  $i \in N$  at time  $\tau \in [t_0, T]$  equaling

$$\begin{aligned}
 B_i(\tau) = & - \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!} \{ [W_t^{(\tau)K}(\tau, x_K^{\tau*})|_{t=\tau}] - \\
 & [W_t^{(\tau)K \setminus i}(\tau, x_{K \setminus i}^{\tau*})|_{t=\tau}] + ([W_{x_N^{\tau*}}^{(\tau)K}(t, x_K^{\tau*})|_{t=\tau}] - \\
 & [W_{x_N^{\tau*}}^{(\tau)K \setminus i}(t, x_{K \setminus i}^{\tau*})|_{t=\tau}]) \times f^N[\tau, x_N^{\tau*}, \psi_N^{(\tau)N}(\tau, x_N^{\tau*})] \} + \\
 & \frac{1}{2} \sum_{h, \zeta=1}^m \Omega_K^{h\zeta}(\tau, x_\tau^*) [W_{x_t^h x_t^\zeta}^{(\tau)K}(t, x_t^*)|_{t=\tau}] - \\
 & \frac{1}{2} \sum_{h, \zeta=1}^m \Omega_{K \setminus i}^{h\zeta}(\tau, x_\tau^*) [W_{x_t^h x_t^\zeta}^{(\tau)K \setminus i}(t, x_t^*)|_{t=\tau}] \}
 \end{aligned}$$

Stochastic environment factor

will lead to the realization of the Condition.



# Future Work

- Design practical protocol based on our incentive framework
- Implementation and validation of our protocol based on Virtual BT platform



# End

- Review
  - Analysis of Tit-for-Tat Strategy
  - Basic Elements of Game Theory
  - Incentive Framework (theoretical)
- Thanks 😊