A new BEMD method based on self-similar feature

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Abstract

This paper presents a new method for texture analysis through Bidimensional Empirical Mode Decomposition (BEMD). Although there have been many filter based methods for texture analysis, problems of non-adaptively and redundancy are still hard to solve. The BEMD is a locally adaptive method and suitable for the analysis of nonlinear or nonstationary signals. The texture image can be decomposed to several IMFs (intrinsic mode functions) by BEMD, which present new characters of the images. But for the BEMD, the boundary interference is a main limit for its application. In this paper, we proposed a new BEMD method based on the self-similar extend method and the neighbor local extremes to reduce the boundary interference. This new method can get a lower orthogonality index (OI) of the IMFs, and the experiment result shown the new method also reduced the computation complex compared to other surface interpolation-based methods. And a denoising algorithm based on the new BEMD is also present in this paper.

1. Introduction

Empirical mode decomposition (EMD), developed by Huang [1], is a data driven processing algorithm, which applies no predetermined filter. The EMD is based on the local characteriristic scale of the data, which is able to perfectly analyze the nonlinear and nonstationary signals. The main processing of the EMD is to decompose the signal to its intrinsic mode functions (IMFs), and then those IMFs are analyzed by the Hilbert transform. This process is also known as Hilbert-Huang transforms (HHT) [1]. The EMD has been used to analyze the two-dimenational signals, for example, the images, which are known as bidimensional EMD (BEMD). BEMD has better quality than Fourier, wavelet and other decomposition algorithms in extracting intrinsic components of textures because of its data driven property [2, 4].

One topic of the BEMD questions is the boundary effect processing, which is to reduce the boundary effect to the intrinsic mode functions (IMFs) [3,4]. There are two ways. One way is to view the image as a long lengthened vector and then apply the one dimensional extended method to solve it [4, 6]. The other way is to extend the image by mirror extending, neural network training or AR model et. [11]. But in some cases, the marked change may also be occur in the image boundary, which make the boundary processing worse, or change the local extrema points. So the boundary processing is still a challenge problem for the BEMD.

Fractal geometry is a kind of new theory for studying non-linear complex systems. Fractal, characterized by self-similar structure, is proposed by Mandelbrot. The self-similar feature means that, irrespective of the complexity of the shape of an object, by looking deeply into its structure, an observer can be the same (or similar) shapes on contractible scales. Such objects exist widely in nature, such as coastlines, contour lines of mountains. With the rapid developments of fractal, fractal geometry has been applied broadly in image processing.

In this paper, we proposed a new BEMD method based on the self-similar extend method and the neighbor local extremes. This new method can reduce the orthogonality index (OI) of the IMFs, and the experiment result shown the new method also reduced the computation complex compared to other surface interpolation based methods. In section 4, based on the new BEMD, an image denoising algorithm is proposed.

2. Review of BEMD

2.1 1-D Empirical Mode Decomposition (EMD)

EMD is first proposed by Huang et al. [1] for the processing of non-stationary functions. The tool decomposes signals into components called Intrinsic Mode Functions (IMFs) satisfying the following two conditions:

(a).The numbers of extrema and zero-crossings must either equal or differ at most by one;

(b).At any point, the mean value of the envelope defined by the local maxima and the envelope by the local minima is zero.

Huang [1] have also proposed an algorithm called 'sifting' to extract IMFs from the original signal *f*(*t*) as follows:

$$f(t) = \sum_{i=1}^{N} I_i(t) + r_N(t) \qquad (1)$$

Where $I_i(t)$, i=1,…,N are IMFs and $r_N(t)$ is the residue.

Although the discussions about EMD lack concrete theoretical foundation until now, there have been some facts demonstrated by experiments. The most attractive one among the facts is that EMD acts as a dyadic filter band for 1-D fractional Gaussian noise and the quality of the obtained Intrinsic Mode Functions is related to the Hurst exponent [5].

2.2 The bidimensional EMD (BEMD)

The bidimensional EMD (BEMD) process is conceptually the same as the one dimension EMD, except that the curve fitting of the maxima and minima envelope now becomes a surface fitting exercise and the identification of the local extrema is performed in space to take into account for the connectivity of the points.

The main process of the BEMD can be described as:

(a).Locate the maximum and minimum points in the image *I(k)*;

(b).Interpolation the surface between the all maxima (resp. munima) to build the envelope *Xmax(k)* and *Xmin(k)*;

(c).Compute the mean envelope function

$$Xm(k) = (Xmax(k) + Xmin(k))/2; \qquad (2)$$

(d).Update the *I(k)=I(k-1)-Xm(k)* ;

(e).Check the stopping criterion

$$SD = \frac{1}{N}\sum_{k=0}^{K}\frac{(I_{i,j-1}(k) - I_{i,j}(k))^2}{I_{i,j-1}^2(k)} \qquad (3)$$

if SD is larger than a threshold ε, repeat the steps (a)-(e) with *I(k)* as the input, other wise, *I(k)* is an IMF *d(k)*;

(f).Update the residual *I(k)=I(k-1)-d(k)*;

(g).Input the *I(k)* to steps(a)-(e) until it can not be decomposed, and the last residual *I(k)=r(n)*.

After the BEMD, the decomposition of the image can be written as following form:

$$I(n) = \sum_{k=1}^{K} d_k(n) + r(n) \qquad (4)$$

The *d(k)* is the IMFs (intrinsic mode functions) of the images, and *r(n)* is the residual function.

3. The new BEMD details

With the intension of some difficult in implement BEMD, we used some methods to improve the BEMD. The local extrema are detected based on its neighbor and the extended parts are rebuilt based on self-similarity.

3.1 Local extrema detection

Detection the local extrema means finding the local maxima and minima points from given images. In the normal BEMD methods [1,4], the mathematical morphology method is used to local the extrema, but we find the extrema points will be reduced fast. It means that, after two or three surface interpolations, the image will be too smooth to local any significative extrema points. Neighbor location method [7] is used to detect the extrema in our method.

Definition 1: f[i,j] is a maximum (or. minimum) if it is larger (or. lower) than the value of *f* at the nearest neighbors of *[i,j]*.

Let *X* be an *M*×*N* 2D matrix represented by

$$X = \begin{bmatrix} x_{11}x_{12}\cdots x_{1N} \\ x_{21}x_{22}\cdots x_{2N} \\ \vdots \;\; \vdots \;\; \cdots \;\; \vdots \\ x_{M1}x_{M2}\cdots x_{MN} \end{bmatrix} \qquad (5)$$

x_{mn} is the element of *X* located in the *m*th row and *n*th column.

Let the window size for local extrema determination be *(2w+1)×(2w+1)*, Then,

$$x_{mn} = \begin{cases} Local\ Maximum & if\ x_{mn} > x_{ij} \\ Local\ Minimum & if\ x_{mn} < x_{ij} \end{cases} \qquad (6)$$

Where

$$x_{ij} = \{x \mid (m-w):i:(m+w), (n-w):j:(n+w)\}$$
$$i \neq m,\ j \neq n \qquad (7)$$

From the experimental, we find *3×3* window results in an optimum extrema map for a given image. The larger windows are also used in some conditions to reduce the computation, but as the mathematical morphology method, the extrema points will be reduced fast.

3.2 Surface interpolation method

Another difficulty in the BEMD comes from generating a smooth fitting surface to the identified maxima and minima. There are several interpolation methods for BEMD. Nunes [2, 8] used the radial basis function (RBF) for surface interpretation. Linderhed [9] used the spline for surface interpretation to develop two-dimensional EMD data. Damerval [10] used a third way based on Delaunay triangulation to obtain an upper surface and a lower surface.

Delaunay triangulation can effectively reduce the interpolation computation. Our interpolation method is based on a Delaunay triangulation. The subset Dmax (resp. Dmin) of D is corresponds to maxima (resp. minima) for f and then on RBF interpolation on triangles as explained in [8]. Delaunay triangulation is well adapted to the interpolation of scattered data for the set Dmax (resp. Dmin).

3.3 Self-similar for BEMD Boundary Processing

A self-similar object is exactly or approximately similar to a part of itself, which means the whole has the same shape as one or more of the parts. Many objects in the real world are statistically self-similar: parts of them show the same statistical properties at many scales. Self-similarity is a typical property of fractals.

A compact topological space X is self-similar if there exists a finite set S indexing a set of non-subjective homeomorphisms $\{f_s\}_{s\in S}$ for which

$$X = \bigcup_{s\in S} f_s(X) \tag{8}$$

If $X \subset Y$, we call X self-similar if it is the only non-empty subset of Y such that the equation above holds for $\{f_s\}_{s\in S}$. We call

$\Gamma = (X, S, \{f_s\}_{s\in S})$ as *self-similar structure*.

The self-similar feature means that, irrespective of the complexity of the shape of an object, by looking deeply into its structure, an observer can be the same (or similar) shapes on contractible scales. In the boundary process, we use this self-similar property to build the extend boundary. The basic idea is: the extend part can find a self-similar part in the original image.

The concrete algorithm is as follows:

Assume the size of original image I is $N\times N$. The size of the extend block is $M\times M$. After extending, the extended image is $(N+2M)\times(N+2M)$ with middle $N\times N$ block the original image. The original image I is divided to $M\times M$ size blocks. For each extended block $part_e$, its three neighbor blocks in the original image are defined as its neighbor blocks $part_n$. And then in the image I, find the blocks which are the most similar to the $part_n$. The similar judgment criterion is based on the MAD (Mean Absolute Difference) for representing the distances different between boundary blocks and the matched blocks. At last, the block with most similar neighbor blocks is used as the extend block.

After the self-similar based extension boundary processing, the boundary interference of the BEMD will be reduced, and the IMF components is more significant.

4. Image denoising based on the new BEMD

The EMD has found a vast number of diverse applications such as biomedical [13], watermarking [14], and audio processing [15]. A more generalized task in which EMD can prove useful is image denoiseing. [12]

The EMD used for denoising tool were developed both by Flandrin[17] and Wu[16], using the statistical analysis model. The energy of the IMFs of a noise-only signal with certain characteristics is known, then in actual cases of signals comprising both information and noise following the specific characteristics, a significant discrepancy between the energy of a noise-only IMF and the corresponding noisy-signal IMF indicates the presence of useful information.[12]

In this part, we will show an image denoisig method based on the new BEMD. First, the noised image is decomposed by the BEMD, and then, the IMFs are denoised by the denoising factor by different paramters, at last, the denoised components are composed.

This denoising factor is less than 1 and will decrease when the absolute value of the coefficients increases. This algorithm considers energy distribution property of the decomposition of the image globally, and obtains a better denoising result. The realization process is as follows:

$$F(x,y) = \begin{cases} w(x,y) & |w(x,y) \geq 2\sigma| \\ 0 & |w(x,y) \leq |aver|| \\ w(x,y)\times k & else \end{cases} \tag{5}$$

Where *w(x,y)* denotes the coefficient, *F(x,y)* denotes the coefficient gained after denoising; *σ*, *aver* indicate the variance and the mean value from different decomposition levels and different directions [18].

5. Experimental results

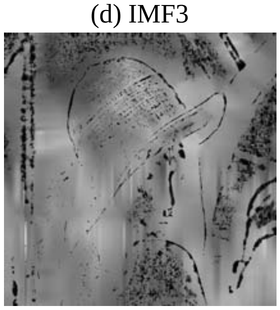
5.1 the new BEMD decomposition result

In this section, we present the experimental results of the image to decompose by using the self-similar extension and the neighbor local extremes.

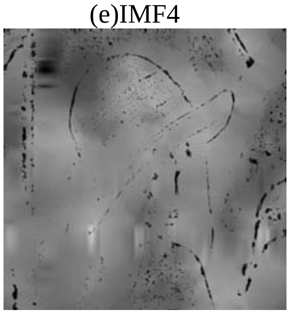
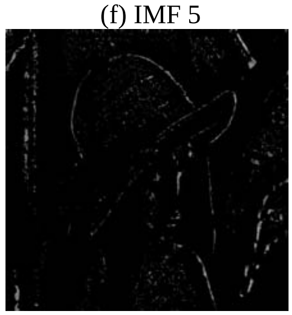
Image can be regarded as nonlinear signal in two-dimensional. The BEMD can decompose the signals adaptively so it is suitable for analyzing the image. The proposed BEMD create a filter bank when applied to the texture images. Figure 1 shows a decomposition result of image.


(a) Original image (b) IMF1


(c)IMF2 (d) IMF3

(e)IMF4 (f) IMF 5

(g)IMF 6 (h) Residue

Figure1 Original image and its decomposition IMFs

The orthogonaltiy index (OI), has been proposed for IMFs in [1], the extended formula for two dimensional is defined as follows:

$$OI = \sum_{x=1}^{M}\sum_{y=1}^{N}\left(\sum_{i=1}^{K+1}\sum_{j=1}^{K+1}\frac{C_i(x,y)C_j(x,y)}{\sum C^2(x,y)}\right) \qquad (9)$$

A low value of the OI indicated a good decomposition in terms of local orthogonality among the IMFs [4,7]. The texture image's OI by our proposed method and some other methods are shown in the Table 1.

Methods	OI	Consuming time (s)
Nunes[8]	0.0033	68.52
Linderhed [9]	0.0029	60.51
Our method	0.0025	46.59

Table 1. Compared result of Orhogonaltiy index (OI) and the consuming time

As the Table 1 shown, our method can reduce the OI, which improve the BEMD method, and the consuming time is also reduced.

5.2 The new-BEMD based image denoising

In this part, results of the denoising method discussed in section 4 are present.

The traditional denoising algorithms, such as median filter and Wiener filter, can reduce the Gaussian noise and salt & pepper noise. But the PSNR of the denoise result is still not idea; the noise is still in the image. BEMD method decomposed the image to many IMFs, which are basically the image portion contained in each adaptive subband. By use of the factor, the noise can be removed scale by scale.[12]


(a) noise image (PSNR: 18.718) (b) median filter


(c) Wiener filter (d) Nunes[8]+ factor

(e) Our approach

Figure 2. noise image and compared denoised image

The result shown that based on the BEMD and denoise factor, the noise can be reduced more effetely than the traditional methods. And the new BEMD's denoise result is also better than the Nunes' BEMD.

Output PSNR / Input PSNR	Median filter	Wiener filter	Nunes[8]+ factor	BEMD + factor
18.718	24.323	24.59	26.287	29.31
16.146	22.947	22.61	24.599	27.307
14.508	21.66	21.352	24.402	26.503

Table 2 denoising result with output PSNR

6. Conclusion and future work

Aimining for the boudary effects of BEMD, a new BEMD method is proposed, which is based on the self-similar feature and the neighbor local extremes. This new method can reduce the orthogonality index (OI) of the IMFs, and the experiment result shown the new method also reduced the computation complex compared to other surface interpolation based BEMD methods.

Based on the new BEMD method, we proposed a image denoising method, compare with the normal denoising algorithm, the BEMD-based method can achieve a good result. In the future, we will focus on developing a better BEMD method to reduce the OI and more research about the BEMD application in image processing and pattern recognition.

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