# Supplemental Material

## Appendix A: Proof of Theorem 1

Without loss of generality, we assume that the data item has been broadcast at ticks 0 and $T$. Let $l$ be the broadcast tick. Given a fixed number of broadcast instances, $n+1$, during the interval $[0, T \cdot l]$, we consider the broadcast schedule for $[0, T \cdot l]$. Suppose the item is broadcast at times

$$0 = t_0 < t_1 < t_2 < \cdots < t_n = T \cdot l.$$

Let

$$\tau_i = t_i - t_{i-1}, \qquad (i = 1, 2, \cdots, n)$$

be the duration between two consecutive broadcast instances $t_i$ and $t_{i-1}$.

According to Equation (1), the total request drop rate during $[0, T \cdot l]$ is given by

$$\frac{1}{T \cdot l}\Big(\int_0^{\tau_1} F(\tau_1 - t)\,dt + \int_0^{\tau_2} F(\tau_2 - t)\,dt + \cdots + \int_0^{\tau_n} F(\tau_n - t)\,dt\Big). \tag{11}$$

We first prove that for any $i \neq j$,

$$\int_0^{\tau_i} F(\tau_i - t)\,dt + \int_0^{\tau_j} F(\tau_j - t)\big)\,dt \ \geq 2 \cdot \int_0^{\frac{\tau_i+\tau_j}{2}} F(\frac{\tau_i + \tau_j}{2} - t)\,dt.$$

Without loss of generality, suppose $\tau_i \geq \tau_j$. Since $F(x)$ is a non-decreasing function, we have

$$F(x) \ \geq \ F(x - \frac{\tau_i - \tau_j}{2}).$$

Therefore,

$$\int_{\frac{\tau_i+\tau_j}{2}}^{\tau_i} F(x)\,dx \ \geq \ \int_{\frac{\tau_i+\tau_j}{2}}^{\tau_i} F(x - \frac{\tau_i - \tau_j}{2})\,dx,$$

and

$$\int_{\frac{\tau_i+\tau_j}{2}}^{\tau_i} F(x)\,dx \ \geq \ \int_{\tau_j}^{\frac{\tau_i+\tau_j}{2}} F(x)\,dx.$$

As a result,

$$\int_0^{\tau_i} F(x)\,dx + \int_0^{\tau_j} F(x)\,dx \ \geq \ 2 \cdot \int_0^{\frac{\tau_i+\tau_j}{2}} F(x)\,dx. \tag{12}$$

Since for any $\tau$,

$$\int_0^{\tau} F(\tau - t)\,dt = \int_0^{\tau} -F(\tau - t)\,d(\tau - t) = \int_{\tau}^{0} -F(x)\,dx \ = \ \int_0^{\tau} F(x)\,dx,$$

it follows from (12) that

$$\int_0^{\tau_i} F(\tau_i - t)\,dt + \int_0^{\tau_j} F(\tau_j - t)\,dt \;\geq 2 \cdot \int_0^{\frac{\tau_i+\tau_j}{2}} F(\frac{\tau_i + \tau_j}{2} - t)\,dt.$$

This implies the lowest drop rate is achieved when $\tau_1 = \tau_2 = \cdots = \tau_n$, since otherwise, we can obtain an equal or a lower drop rate by replacing two different intervals $\tau_i$ and $\tau_j$ each with $\frac{\tau_i+\tau_j}{2}$. Hence, the theorem is proven. □

## Appendix B: Proof of Theorem 2

Since $\sum_{i=1}^{N} \frac{1}{s_i} = \frac{1}{l}$, we have $\sum_{i=1}^{N} \frac{1}{s_i} - \frac{1}{l} = 0$. Let

$$L(s_1, s_2, \cdots, s_N, \gamma) = \eta(s_1, s_2, \cdots, s_N) + \gamma\big(\sum_{i=1}^{N} \frac{1}{s_i} - \frac{1}{l}\big). \tag{13}$$

It is obvious that minimizing $\eta(s_1, s_2, \cdots, s_N)$ is equivalent to minimizing $L(s_1, s_2, \cdots, s_N, \gamma)$ defined above.

Substituting (4) for $\eta(s_1, s_2, \cdots, s_N)$, we rewrite (13) as follows:

$$L(s_1, s_2, \cdots, s_N, \gamma) = 1 - \sum_{i=1}^{N} \frac{p_i M(1 - e^{-\frac{s_i}{M}})}{s_i} + \gamma\big(\sum_{i=1}^{N} \frac{1}{s_i} - \frac{1}{l}\big). \tag{14}$$

Differentiate (14) by $s_i$ and equal it to zero, we obtain

$$p_i M(1 - \frac{s_i}{M} e^{-\frac{s_i}{M}} - e^{-\frac{s_i}{M}}) - \gamma = 0. \tag{15}$$

It is easy to infer that for any $s_i$ value satisfying (15), the partial derivative $\frac{\partial^2 L}{\partial s_i^2} > 0$ and the partial derivative $\frac{\partial^2 L}{\partial s_i \partial s_j} = 0$. Therefore, the set of inter-broadcast durations $(s_1, s_2, \cdots, s_N)$ satisfying (15) minimizes $L(s_1, s_2, \cdots, s_N, \gamma)$ and thus $\eta(s_1, s_2, \cdots, s_N)$. Hence, the theorem is proven.

## Appendix C: Proof of Lemma 1

Assume on the contrary that there exist two items $i$ and $j$ such that $p_i > p_j$, $s_j^* \leq L$ and $s_i^* > L$. Consider the set of inter-broadcast durations $(s_1', s_2', \cdots, s_N')$ where $s_i' = s_j^*$, $s_j' = s_i^*$, and $\forall k \neq i, j, s_k' = s_k^*$. Let $P$ be the drop rate of $(s_1^*, s_2^*, \cdots, s_N^*)$. The drop rate of

$(s'_1, s'_2, \cdots, s'_N)$ is given by

$$\begin{aligned} P' &= P - p_i\eta(s^*_i) - p_j\eta(s^*_j) + p_i\eta(s'_i) + p_j\eta(s'_j) \\ &= P - p_i(1 - \frac{L}{2s^*_i}) - p_j\frac{s^*_j}{2L} + p_i\frac{s^*_j}{2L} + p_j(1 - \frac{L}{2s^*_i}) \\ &= P + (p_j - p_i)\big(1 - \frac{L}{2s^*_i} - \frac{s^*_j}{2L}\big) \\ &= P + (p_j - p_i)\frac{L(s^*_i - L) + s^*_i(L - s^*_j)}{2s^*_i L} \quad < P, \end{aligned}$$

which contradicts with the optimality of inter-broadcast durations $(s^*_1, s^*_2, \cdots, s^*_N)$.

Hence, the lemma is proven. □

## Appendix D: Proof of Lemma 2

Let $(s^*_1, s^*_2, \cdots, s^*_N)$ be a set of inter-broadcast durations producing the lowest request drop rate. It follows from Lemma 1 that there exists an identification item $I$ such that $\forall i \le I$, $s^*_i \le L$, and $\forall i > I$, $s^*_i > L$.

First, we prove that all non-infinity inter-broadcast durations in $s^*_{I+1}, s^*_{I+2}, \cdots, s^*_N$ are associated with data items of equal access probabilities. Assume on the contrary that there exist two non-infinity durations $s^*_i$ and $s^*_j$ $(j > i > I)$ such that $p_i > p_j$ (remember that item indexes are numbered in decreasing order of access probability). Consider the set of inter-broadcast durations $(s'_1, s'_2, \cdots, s'_N)$ where $s'_i = \frac{1}{\frac{1}{s^*_i}+\Delta}$, $s'_j = \frac{1}{\frac{1}{s^*_j}-\Delta}$, and $\Delta = min\big(\frac{1}{2L} - \frac{1}{2s^*_i}, \frac{1}{2s^*_j}\big)$ and $\forall k \ne i, j, s'_k = s^*_k$. It is easy to verify that $\sum_{k=1}^{N} \frac{1}{s^*_k} = \sum_{k=1}^{N} \frac{1}{s'_k}$, $s'_i > L$, and $s'_j > L$. Let $P$ be the drop rate of $(s^*_1, s^*_2, \cdots, s^*_N)$. The drop rate of $(s'_1, s'_2, \cdots, s'_N)$ is given by

$$\begin{aligned} P' &= P - p_i\eta(s^*_i) - p_j\eta(s^*_j) + p_i\eta(s'_i) + p_j\eta(s'_j) \\ &= P - p_i(1 - \frac{L}{2s^*_i}) - p_j(1 - \frac{L}{2s^*_j}) + p_i(1 - \frac{L(\frac{1}{s^*_i} + \Delta)}{2}) + p_j(1 - \frac{L(\frac{1}{s^*_j} - \Delta)}{2}) \\ &= P - (p_i - p_j)\cdot\frac{L}{2}\cdot\Delta \quad < P, \end{aligned}$$

which contradicts with the optimality of inter-broadcast durations $(s^*_1, s^*_2, \cdots, s^*_N)$.

Moreover, it is easy to infer that the non-infinity inter-broadcast durations in $s^*_{I+1}, s^*_{I+2}, \cdots, s^*_N$ are associated with data items of access probability $p_{I+1}$. This is because otherwise, $s^*_{I+1}$ must be infinity and therefore, exchanging a non-infinity duration with that of item $I + 1$ would result in a lower drop rate.

So far, we have shown that given an optimal set of inter-broadcast durations $(s_1^*, s_2^*, \cdots, s_N^*)$, there exists an index $J$ $(I \le J \le N)$ such that $\forall i \le I$, $s_i^* \le L$; $\forall I+1 \le i \le J$, $p_i = p_{I+1}$, $s_i^* > L$; and $\forall i > J$, $s_i^* = \infty$, where $I$ is the identification item.

If $I = J$, $(s_1^*, s_2^*, \cdots, s_N^*)$ is an optimal set of durations following claims (i), (ii) and (iii).

Otherwise, if $I < J$, it is easy to show that the set of durations $(s'_1, s'_2, \cdots, s'_N)$ where

- $\forall i \le I$, $s'_i = s_i^*$;
- $\forall I+1 \le i \le I + \lfloor \sum_{i=I+1}^{J} \frac{L}{s_i^*} \rfloor$, $s'_i = L$;
- $s'_{I+\lfloor \sum_{i=I+1}^{J} \frac{L}{s_i^*} \rfloor + 1} = \frac{1}{\sum_{i=I+1}^{J} \frac{L}{s_i^*} - \lfloor \sum_{i=I+1}^{J} \frac{L}{s_i^*} \rfloor \frac{1}{L}}$;
- $\forall I + \lfloor \sum_{i=I+1}^{J} \frac{L}{s_i^*} \rfloor + 2 \le i \le N$, $s'_i = \infty$,

also produces the lowest drop rate and satisfies claims (i), (ii) and (iii), where $I' = I + \lfloor \sum_{i=I+1}^{J} \frac{L}{s_i^*} \rfloor$ is taken as the identification item.

Hence, the lemma is proven. □

## Appendix E: Proof of Lemma 3

We prove an even stronger claim: each set of values $(s_1, s_2, \cdots, s_I)$ where $\sum_{i=1}^{I} \frac{1}{s_i} = \frac{f}{l}$ and $\forall m+1 \le i \le I, s_i \le L$ produces a value of (7) higher than or equal to that of durations $(s_1^*, s_2^*, \cdots, s_m^*, L, L, \cdots, L)$. Note that the definition of $m$ ensures $\forall 1 \le i \le m$, $s_i^* \le L$, and $f \ge \frac{I \cdot l}{L}$ implies $m \ge 1$.

The value of (7) for $(s_1^*, s_2^*, \cdots, s_m^*, L, L, \cdots, L)$ is given by

$$\frac{1}{2L(\frac{f}{l} - \frac{I-m}{L})} (\sum_{i=1}^{m} \sqrt{p_i})^2 + \sum_{i=m+1}^{I} \frac{p_i}{2}.$$

By applying the Lagrange multiplier method [11], the lowest value of (7) given $s_{m+1}, s_{m+2}, \cdots, s_I \le L$ is:

$$\frac{1}{2L(\frac{f}{l} - \sum_{i=m+1}^{I} \frac{1}{s_i})} (\sum_{i=1}^{m} \sqrt{p_i})^2 + \sum_{i=m+1}^{I} \frac{p_i s_i}{2L}.$$

Thus, it is sufficient to prove

$$
\begin{aligned}
& \frac{\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2}{2L(\frac{f}{l}-\sum_{i=m+1}^{I}\frac{1}{s_i})}+\sum_{i=m+1}^{I}\frac{p_i s_i}{2L} \geq \frac{\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2}{2L(\frac{f}{l}-\frac{I-m}{L})}+\sum_{i=m+1}^{I}\frac{p_i}{2} \\
\iff & \sum_{i=m+1}^{I} p_i - \sum_{i=m+1}^{I}\frac{p_i s_i}{L} \leq \frac{\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2}{L(\frac{f}{l}-\sum_{i=m+1}^{I}\frac{1}{s_i})} - \frac{\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2}{L(\frac{f}{l}-\frac{I-m}{L})} \\
\iff & \sum_{i=m+1}^{I} p_i(1-\frac{s_i}{L}) \leq \frac{\left(m-I+\sum_{m+1}^{I}\frac{L}{s_i}\right)\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2}{L^2(\frac{f}{l}-\frac{I-m}{L})(\frac{f}{l}-\sum_{i=m+1}^{I}\frac{1}{s_i})}.
\end{aligned}
$$

It follows from the definition of $m = max\{x \mid \frac{\sum_{j=1}^{x}\sqrt{p_j}}{(\frac{f}{l}-\frac{I-x}{L})\sqrt{p_x}} \leq L\}$ (in Lemma 3) that

$$
\begin{aligned}
& \frac{\sum_{i=1}^{m+1}\sqrt{p_i}}{(\frac{f}{l}-\frac{I-m-1}{L})\sqrt{p_{m+1}}} > L \\
\iff & \sqrt{p_{m+1}} < \frac{\sum_{i=1}^{m}\sqrt{p_i}+\sqrt{p_{m+1}}}{L(\frac{f}{l}-\frac{I-m}{L})+1} \\
\iff & \sqrt{p_{m+1}} < \frac{\sum_{i=1}^{m}\sqrt{p_i}}{L(\frac{f}{l}-\frac{I-m}{L})} \\
\iff & p_{m+1} < \frac{\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2}{L^2(\frac{f}{l}-\frac{I-m}{L})^2},
\end{aligned}
$$

and

$$
s_i \leq L \iff \frac{1}{L} \leq \frac{1}{s_i} \iff \frac{I-m}{L} \leq \sum_{i=m+1}^{I}\frac{1}{s_i}.
$$

Therefore,

$$
\begin{aligned}
\sum_{i=m+1}^{I} p_i(1-\frac{s_i}{L}) & \leq p_{m+1}\sum_{i=m+1}^{I}(1-\frac{s_i}{L}) \\
& \leq \frac{\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2}{L^2(\frac{f}{l}-\frac{I-m}{L})^2}\sum_{i=m+1}^{I}(1-\frac{s_i}{L}) \\
& \leq \frac{\left(\sum_{i=1}^{m}\sqrt{p_i}\right)^2\left(I-m-\sum_{i=m+1}^{I}\frac{s_i}{L}\right)}{L^2(\frac{f}{l}-\frac{I-m}{L})(\frac{f}{l}-\sum_{i=m+1}^{I}\frac{1}{s_i})}.
\end{aligned}
$$

Since

$$
\begin{aligned}
\frac{L}{s_i}+\frac{s_i}{L} \geq 2 & \iff \sum_{i=m+1}^{I}(\frac{L}{s_i}+\frac{s_i}{L}) \geq 2(I-m) \\
& \iff I-m-\sum_{i=m+1}^{I}\frac{s_i}{L} \leq m-I+\sum_{i=m+1}^{I}\frac{L}{s_i},
\end{aligned}
$$

it follows that

$$\begin{aligned}\sum_{i=m+1}^{I} p_i(1-\frac{s_i}{L}) &\le \frac{\big(\sum_{i=1}^{m}\sqrt{p_i}\big)^2\big(I-m-\sum_{i=m+1}^{I}\frac{s_i}{L}\big)}{L^2(\frac{f}{l}-\frac{I-m}{L})(\frac{f}{l}-\sum_{i=m+1}^{I}\frac{1}{s_i})} \\ &\le \frac{\big(m-I+\sum_{m+1}^{I}\frac{L}{s_i}\big)\big(\sum_{i=1}^{m}\sqrt{p_i}\big)^2}{L^2(\frac{f}{l}-\frac{I-m}{L})(\frac{f}{l}-\sum_{i=m+1}^{I}\frac{1}{s_i})}.\end{aligned}$$

Hence, the lemma is proven. □